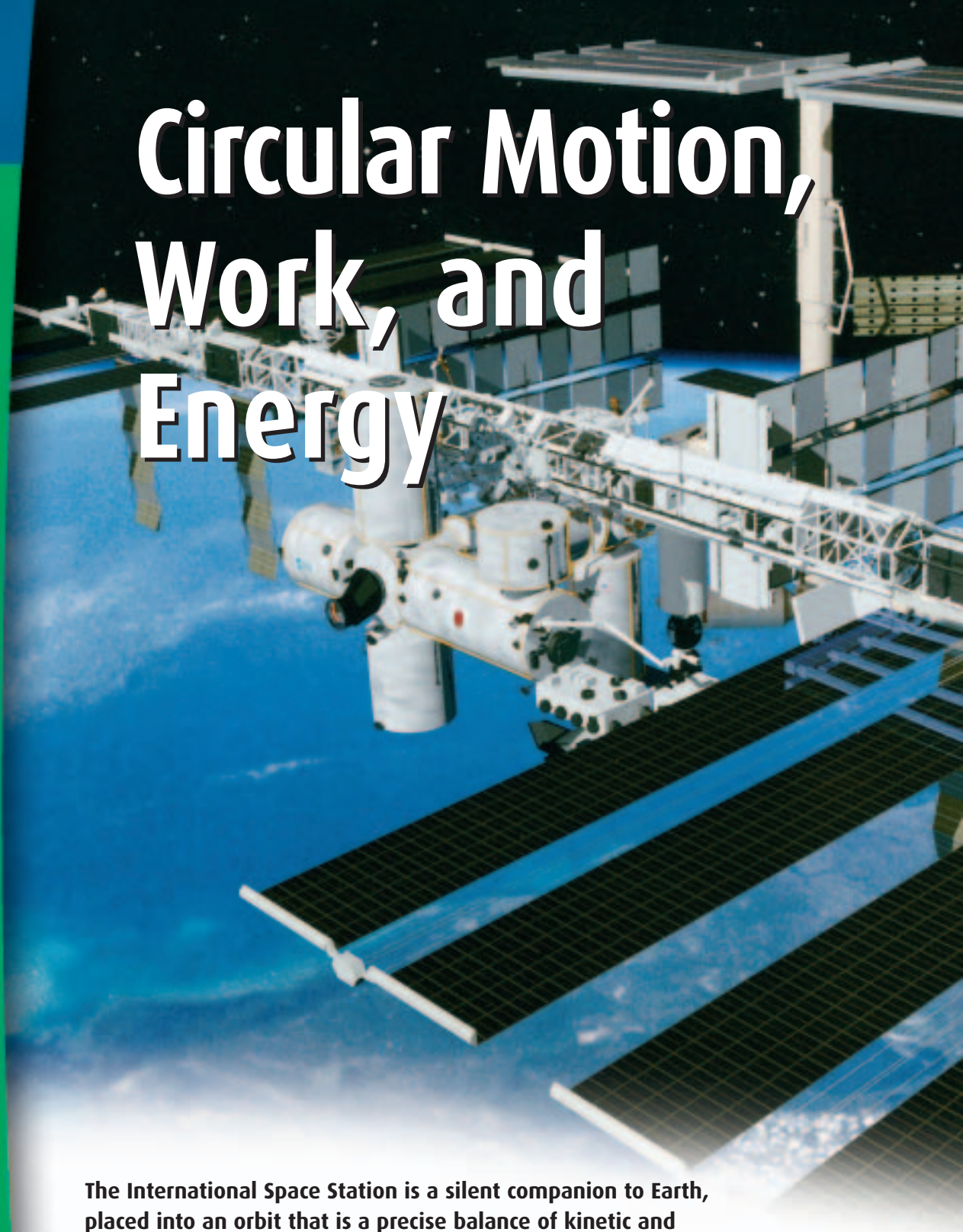
UNIT

III

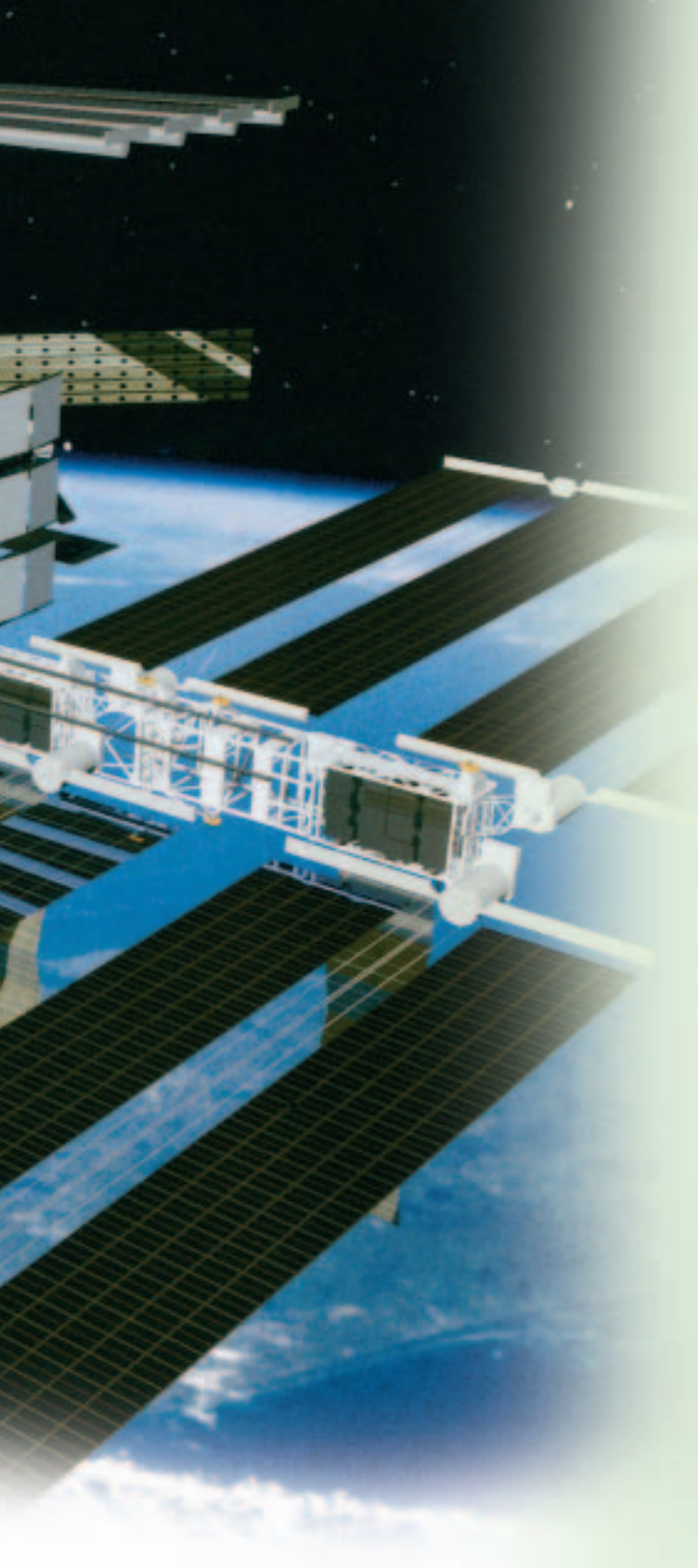
Circular Motion, Work, and Energy



The International Space Station is a silent companion to Earth, placed into an orbit that is a precise balance of kinetic and gravitational potential energies. The International Space Station stays in orbit because physicists and engineers applied the laws of physics for circular motion and conservation of energy to determine the satellite's speed and height above Earth.

eWEB

To learn more about the types of satellites placed in orbit, and the paths that they take, follow the links at www.pearsoned.ca/school/physicssource.



Unit at a Glance

CHAPTER 5 Newton's laws can explain circular motion.

- 5.1 Defining Circular Motion
- 5.2 Circular Motion and Newton's Laws
- 5.3 Satellites and Celestial Bodies in Circular Motion

CHAPTER 6 In an isolated system, energy is transferred from one object to another whenever work is done.

- 6.1 Work and Energy
- 6.2 Mechanical Energy
- 6.3 Mechanical Energy in Isolated and Non-isolated Systems
- 6.4 Work and Power

Unit Themes and Emphases

- Energy, Equilibrium, and Systems
- Nature of Science
- Scientific Inquiry

Focussing Questions

This unit focusses on circular motion, work, and energy. You will investigate the conditions necessary to produce circular motion and examine some natural and human examples. You will consider energy, its transfer, and how it interacts with objects. As you study this unit, consider these questions:

- What is necessary to maintain circular motion?
- How does an understanding of conservation laws contribute to an understanding of the Universe?
- How can mechanical energy be transferred and transformed?

Unit Project

Building a Persuader Apparatus

- When you have finished this unit, you will understand how energy is transferred when objects interact. You will be able to use this understanding in the design and construction of a persuader apparatus that is able to protect its passenger from injury in different types of collisions.

CHAPTER

5

Newton's laws can explain circular motion.

Key Concepts

In this chapter, you will learn about:

- uniform circular motion
- planetary and satellite motion
- Kepler's laws

Learning Outcomes

When you have completed this chapter, you will be able to:

Knowledge

- describe uniform circular motion as a special case of two-dimensional motion
- explain centripetal acceleration and force
- explain, quantitatively, the relationships among speed, frequency, period, and radius for circular motion
- explain, qualitatively, uniform circular motion using Newton's laws of motion
- explain, quantitatively, the motion of orbiting bodies by using circular motion to approximate elliptical orbits
- predict the mass of a celestial body from orbital data
- explain, qualitatively, how Kepler's laws were used to develop Newton's universal law of gravitation

Science, Technology, and Society

- explain the process of scientific inquiry
- illustrate how science and technology are developed to meet society's needs and expand human capabilities
- analyze circular motion in daily situations

If humans hadn't invented the wheel — the first circular motion machine — it's hard to imagine what life would be like. The number of devices that make use of the properties of circular motion is almost too large to count. Bicycles, gears, drills, transmissions, clutches, cranes, watches, and electric motors are just a few examples.

The wheel and the many technologies derived from it are a uniquely human creation (Figure 5.1). But the principles of circular motion have always existed in nature. In fact, as you read this, you are spinning around in a large circle as Earth rotates on its axis. In Alberta, your speed is approximately 1000 km/h because of this rotation. At the same time, you are flying through space at an amazing speed of approximately 107 000 km/h as Earth revolves around the Sun. And you thought your car was fast!

How is circular motion unique? What are the properties of objects moving with circular motion? In this chapter, you will explore the physics of circular motion that define and control these many technologies, as well as the motion of the planets in the solar system.

▲ **Figure 5.1**

5-1 QuickLab

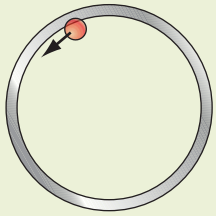
Characteristics of Circular Motion

Problem

In what direction will an object moving in a circular path go when released?

Materials

a marble
a circular barrier (e.g., rubber tubing, embroidery hoop)



▲ Figure 5.2

Procedure

1. Place the circular barrier on an unobstructed section of a table or the floor, then place the marble against the inside rim of the barrier (Figure 5.2).
2. You will be rolling a marble around inside the barrier. Before you do, predict where you think the marble will go when you remove the barrier. Now roll the marble around the inside rim of the barrier.
3. As the marble is rolling around the rim, lift the barrier and make a note of the direction the marble rolls. Also pay attention to the position of the marble when you lift the barrier.
4. Sketch the circular path that the marble took inside the barrier, the position of the marble when you lifted the barrier, and the path that it rolled after the barrier was lifted.
5. Repeat steps 3 and 4 several times. Each time release the marble when it is in a different position.

Questions

1. Was your prediction correct? Why or why not?
2. What similarities, if any, exist between your sketches?
3. What conclusions can you draw about the motion of the marble when it was released?
4. In each sketch that you made, draw a line from the centre of the circular motion to the position of the marble when it was released. What is the angle between this line and the path of the marble when it was released?

Think About It

1. Can you predict the position the marble would have to be in for it to move away from your body when it was released?
2. What can you say about the direction of the velocity of the marble at any moment in its circular path?

Discuss your answers in a small group and record them for later reference. As you complete each section of this chapter, review your answers to these questions. Note any changes to your ideas.

info **BIT**

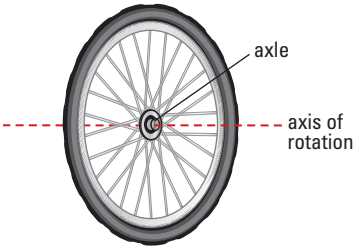
A boomerang has two axes of rotation: one for the spin given to the boomerang as it leaves the hand, and the other for the large circle in which it moves.

5.1 Defining Circular Motion

The bicycle is not the most sophisticated machine ever created, but it does have a lot going for it. It is easy to maintain; it does not require any fuel; it is a very efficient form of transportation; and it is full of parts moving in circular motion. Perhaps most importantly for this lesson, it is easy for us to examine.

When you pedal a bike, the force is transferred through the chain to the **axle** where it turns the wheel (Figure 5.3). The wheel transmits the force to the road, which, according to Newton's third law, pushes back, and the bicycle moves forward. The wheel's axle is referred to as the **axis of rotation** because the entire wheel rotates around this shaft. If the wheels of the bike are moving at a constant speed, they are moving with **uniform circular motion**. That means their rotational speed is uniform. Because the wheel is circular, it can also provide a constant and uniform force to the road. This is important if you want the bike to move forward at a uniform speed.

In the sections that follow, we will restrict our study of circular motion to two dimensions. In other words, the circular motion described in this chapter has only one axis of rotation and the motion is in a plane.



▲ **Figure 5.3** The axle of a wheel is part of the axis of rotation.

axle: shaft on which a wheel rotates

axis of rotation: imaginary line that passes through the centre of rotation perpendicular to the circular motion

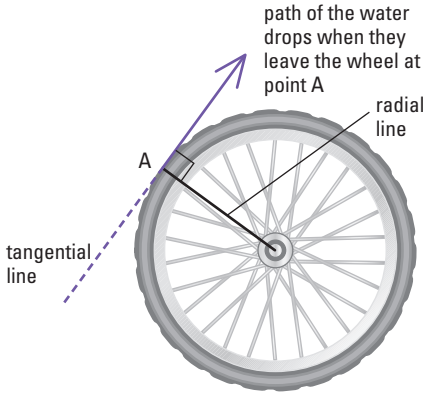
uniform circular motion: motion in a circular path at a constant speed

Speed and Velocity in Circular Motion

If you ride through puddles after a rainfall, you'll come home with muddy water splashed on your back. Why is this? It has to do with the properties of an object moving in circular motion. As the wheel passes through the puddle, some of the water adheres to it.

As the wheel rotates upward out of the puddle, excess water flies off. When it flies off, it moves along a path that is tangential to the wheel. Recall from Unit I that a tangent is a line that touches the circle at only one point and is perpendicular to the radius of the circle. In the case of the bicycle, that is the point where the water drops break free of the wheel. Unless the splashguard is big enough, it will not protect you from all the water that is flying off the wheel (Figure 5.4).

▲ **Figure 5.4** Water that flies off the wheel at point A hits the rider in the back. The direction that the water drops fly is determined by the place where they leave the wheel.



▲ **Figure 5.5** Water leaves the wheel at point A and flies off at a tangent.

A line drawn from point A in Figure 5.5 to the centre of the circle is called a radial line. The radial line and the tangential line are perpendicular to each other when they intersect at the same point on the circle. The tangential line represents the direction that the water drops are moving at any instant. The speed of the water drops is determined by their speed at the wheel.

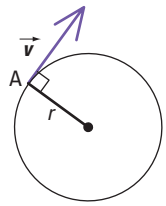
It is worth taking some time to review the difference between speed and velocity. Speed is a magnitude only (a scalar quantity) that does not have a direction. If a wheel is rotating with uniform circular motion, then the speed is constant, even though the wheel is continually turning and the water on the wheel is continually changing direction. Velocity, on the other hand, has a magnitude and a direction: it is a vector quantity. Because of this, even though the wheel is spinning at constant speed, the velocity of the water on the wheel is continually changing as the wheel's direction of motion changes.

It is correct to say that the speed of the water drops on the wheel represents the magnitude of the velocity. Knowing that the object moves in a circular path is often sufficient. However, we must specify the object's velocity if we need to know its speed and direction at any instant in its circular path.

Since we can assume the speed and direction of the water drop are known at any instant, we know its velocity. A velocity vector can be drawn at position A, and the drawing can be simplified as shown in Figure 5.6.

PHYSICS INSIGHT

A line tangent to the circle represents the velocity vector. It is perpendicular to the radial line at that point.



▲ **Figure 5.6** The velocity vector $\vec{v}$ shows the velocity of a particle at any instant as it moves on a circular path.

Concept Check

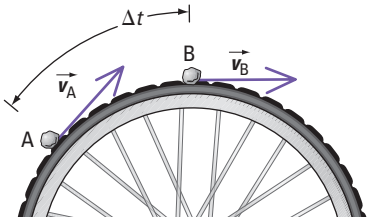
1. Imagine a Frisbee in level flight. Where is its axis of rotation, and what is its orientation?
2. Identify all the axes of rotation in a typical bicycle.

▲ **Figure 5.7** A mountain bike has many axes of rotation.

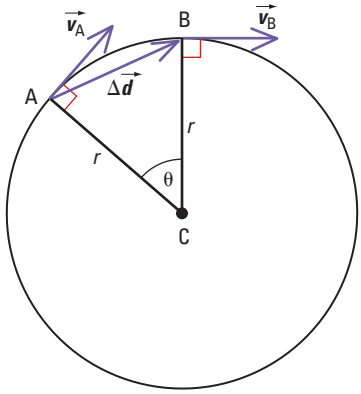
Centripetal Acceleration and Force

Now imagine that you drive over a small pebble that gets lodged in the treads of your bicycle wheel. As you ride, the pebble circles around with the wheel, as shown in Figure 5.8. At one moment it is in position A, and a fraction of a second later, it is in position B. A short time has passed as the pebble moved from point A to point B. This small change in time is written as Δt.

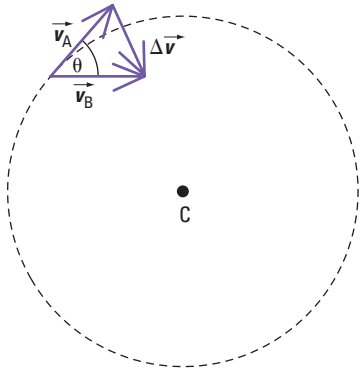
The pebble has experienced a very small displacement from point A to point B, which is written as $\Delta\vec{d}$. The velocities of the pebble at point A and point B are $\vec{v}_A$ and $\vec{v}_B$, respectively (Figure 5.9). C represents the centre of the circle (the axis of rotation).



▲ **Figure 5.8** A pebble caught in the wheel of a bike moves from position A to B in a small time Δt, and experiences a change in velocity.



▲ **Figure 5.9** As the pebble moves from point A to point B, it moves through angle θ and experiences a change in velocity.

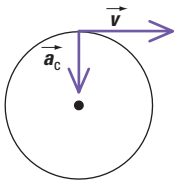


▲ **Figure 5.10** The change in velocity $\vec{v}$ is toward the centre of the circle.

The speed of the pebble does not change, but its direction does. Therefore, its velocity also changes. The change in velocity ($\Delta\vec{v}$) can best be shown by subtracting the velocity at B from the velocity at A (using the rules of graphical vector addition) as shown in Figure 5.10. Angle θ is the same in both Figures 5.9 and 5.10, and the two triangles are similar. Something subtle but significant happens when we subtract the two velocity vectors. The difference between them is designated as $\Delta\vec{v}$, and it is pointing in toward the centre of the circle!

Velocity and Acceleration Toward the Centre

The changing velocity ($\Delta\vec{v}$) represents the change in direction of the object, not its speed. If an object has a changing velocity, then it must be accelerating. Since the changing velocity is pointing inward toward the centre of the circle, the acceleration must also be in that direction (Figure 5.11). It is called **centripetal acceleration** ($\vec{a}_c$). For an object to move with circular motion, it must experience centripetal acceleration. If the circular motion is uniform, then so is the centripetal acceleration.



▲ **Figure 5.11** Although the velocity of an object moving with uniform circular motion is tangential, the centripetal acceleration (and centripetal force) is acting toward the centre of the circle.

centripetal acceleration: acceleration acting toward the centre of a circle

centripetal force: force acting toward the centre of a circle causing an object to move in a circular path

According to Newton's second law, if a mass is accelerating, it must also experience a non-zero net force. This non-zero net force is called the **centripetal force** ($\vec{F}_c$). In our example, the pebble stuck in the wheel is experiencing a force that attempts to pull it toward the centre of the circle. This is the centripetal force.

Why doesn't the pebble actually move toward the centre of the wheel? It does, in a way. Remember: if the pebble were to break free of the tire's grip, it would fly off at a tangent to its circular motion. While it remains stuck in the tire, it is forced to follow the circular path because the centripetal force attempts to pull it toward the centre. In other words, centripetal force is pulling the pebble toward the centre of the circle while at the same time, the pebble is moving off in a direction at a tangent to the circle. The result is the circular path in which it actually moves.

5-2 Inquiry Lab

Required Skills
- Initiating and Planning
- Performing and Recording
- Analyzing and Interpreting
- Communication and Teamwork

Speed and Radius

Question

For an object moving with uniform circular motion, what relationship exists between the radius of its path and its speed? (Assume the object is experiencing a constant centripetal force.)

Hypothesis

State a hypothesis relating the radius and speed. Remember to use an "if/then" statement.

Variables

The variables in this lab are the radius of the circular path, mass of the rubber stopper, mass of the hanging weight, number of revolutions, elapsed time, period, and speed. Read the procedure and identify the controlled, manipulated, and responding variables.

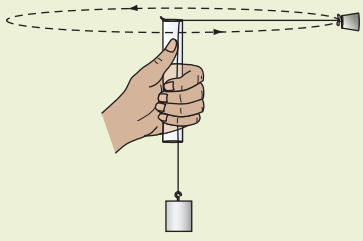
Materials and Equipment

1-hole rubber stopper (mass ≦ 25 g)
1.5 m of string or fishing line
small-diameter plastic tube
100-g mass
metre-stick
felt marker
safety goggles
stopwatch

CAUTION: Remember to swing the rubber stopper over your head and in a place clear of obstructions.

Procedure

1. Copy Table 5.1, shown at the bottom of this page, into your notebook.
2. Secure the rubber stopper to one end of the string.
3. Run the other end of the string through the plastic tube and attach the 100-g mass to it.
4. Hold the end of the string attached to the stopper at the zero mark of a metre-stick laid on a table. The zero mark of the ruler should line up with the centre of the stopper. While holding the stopper in position, pull the string taut along the ruler.
5. With the felt marker, mark the string at 20, 30, 40, 50, and 60 cm.
6. Adjust the string's position so that the 20-cm mark is positioned on the lip of the plastic tube. Record 20 cm in the "Radius" column of the table.
7. Grasp the plastic tube in one hand and pinch the string to the lip of the tube using your thumb or forefinger.
8. Put on your safety goggles. Begin spinning the rubber stopper in a horizontal circle above your head as you release the string. Make sure the 100-g mass is hanging freely (Figure 5.12). At first, you may have to pull the string up or down using your other hand to position the mark as the stopper is spinning.



▲ **Figure 5.12**

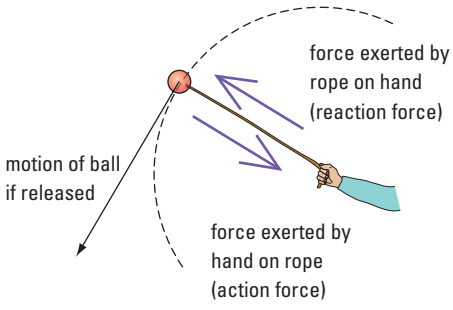
▼ **Table 5.1** Data for 5-2 Inquiry Lab

Radius (m)	Time for 20 Revolutions (s)		Average Time (s)	Period (s)	Speed (m/s)
	Time 1	Time 2			

9. Adjust the rate at which you spin the rubber stopper so that the string does not slip up or down and the mark stays in position at the lip of the tube. Once you have reached a steady rate, your partner can begin timing. Do not pull the string up or down with the other hand.
10. Your partner should time 20 complete revolutions of the rubber stopper using a stopwatch. While your partner does this, be sure to monitor the speed of the stopper so that the mark does not move off the lip. Record the time in the "Time 1" column of the table.
11. Repeat step 10 and record the time in the "Time 2" column of the table.
12. Increase the radius by 10 cm, and record this radius in the "Radius" column of the table. Repeat steps 7 to 12 until all radii are used.

Analysis

1. For each trial, average the two times and place the result in the "Average Time" column.
2. For each trial, divide the average time by the number of revolutions the stopper made. Record the values in the "Period" column.
3. For each trial, determine the speed of the stopper using the equation $v = \frac{2\pi r}{T}$, and record the value in the "Speed" column.
4. Identify the controlled, responding, and manipulated variables.
5. Identify the force that acted as the centripetal force, and determine its value.
6. Plot a graph of velocity versus radius. Remember to plot the manipulated variable on the horizontal axis and the responding variable on the vertical axis.
7. Complete the statement, "The speed varies with …"
8. Was your hypothesis accurate? If not, how can you modify it to reflect your observations from this lab?



▲ **Figure 5.13** The hand exerts a centripetal force on the rope and ball, but feels the reaction force exerted by the rope. This leads to a false impression that the centripetal force is acting outward.

Misconceptions About Centripetal Force

A common misconception is that centripetal force acts radially outward from the centre of a circle. This is not what happens. The change in the velocity of the object is *inward*, and therefore, so is the centripetal acceleration and force. What causes confusion is that when you spin an object in a circle at the end of a rope, you feel the force pulling outward on your hand (Figure 5.13). This outward pull is mistakenly thought to be the centripetal force.

Newton's third law states that for every action there is an equal and opposite reaction. If we apply this law to our example, then the action force is the hand pulling on the rope to make the object move in a circle. This is the centripetal force. The reaction force is the force the rope exerts on your hand. It is outward. This is the force that people often believe is the centripetal force. The rope would not exert a force on your hand unless your hand exerted a force on the rope first.

Another misconception is that centripetal force is a unique and separate force responsible for circular motion. This is not true. It is best to think of centripetal force as a generic term given to any force that acts toward the centre of the circular path. In fact, the translation from Latin of the word centripetal is "centre seeking."

Many different forces could actually be the centripetal force. For example, when a car turns a corner, the frictional force of the tires on the road acts as the centripetal force. If you spin an object around in a horizontal circle on a rope, the tension of the rope is the centripetal force. The force of gravity the Sun exerts on the planets is another example of a centripetal force. Sometimes several forces working together act as a centripetal force. For an object spinning in a vertical circle on a rope, two forces, gravity and tension, work together to act as the centripetal force at the top of the circle. This is because the centripetal force is a net force. It is often convenient to use it in place of the actual force or forces acting toward the centre. Table 5.2 summarizes circular motion quantities and their directions.

PHYSICS INSIGHT

Some texts refer to centrifugal force. This refers to the reaction force that exists as a result of centripetal force. If the centripetal force is removed, there is no centrifugal force.

Concept Check

A pebble caught in the tread of a tire experiences a centripetal force as the tire turns. What force is responsible for it?

eSIM

For an interactive demonstration that explores the relationship among centripetal acceleration, force, and velocity, visit www.pearsoned.ca/school/physicssource.

▼ **Table 5.2** Circular Motion Quantities and Their Direction

Quantity	Direction
Velocity ($\vec{v}$)	tangential to the circle
Centripetal acceleration ($\vec{a}$)	toward the centre
Centripetal force ($\vec{F}_c$ or $\vec{F}_{net}$)	toward the centre
Change in velocity ($\Delta\vec{v}$)	toward the centre

5.1 Check and Reflect

Knowledge

1. Give an example of an object that moves with uniform circular motion and one that moves with non-uniform circular motion.
2. Identify the force acting as the centripetal force in each of the following situations:
   (a) A car makes a turn without skidding.
   (b) A ball is tied to the end of a rope and spun in a horizontal circle.
   (c) The Moon moves in a circular orbit around Earth.

Applications

3. What is the relationship between the speed and radius of an object moving with uniform circular motion if the centripetal force is constant?
4. What is the relationship between the speed and velocity of an object moving with uniform circular motion?

Extensions

5. Imagine pedalling a bike at a constant rate. Describe the motion of the bicycle if the wheels were not circular but oval.
6. Consider the relative speed between a pebble stuck in the tread of a bicycle tire and the ground. Explain why the pebble is not dislodged when it comes in contact with the ground. Suggest a method to dislodge the pebble while still riding the bike.

eTEST

To check your understanding of the definition of circular motion, follow the eTest links at www.pearsoned.ca/school/physicssource.

5.2 Circular Motion and Newton's Laws

info BIT

A backspin on a ball makes it stay aloft longer. A topspin drives the ball downward.

Soccer players put a spin on the ball when they kick it that makes it curve around opposing players. Professional golfers routinely strike the golf ball so that it has a spin that curves it into the wind or prevents it from rolling when it hits the ground. A baseball pitcher can throw a curving fastball at 144 km/h, making the batter's job of hitting the ball much more difficult (Figure 5.14).

▲ **Figure 5.14** Pitchers can put a spin on a ball that causes it to curve. This curve is predictable.

e WEB

To learn more about the effect of putting a spin on a ball, follow the links at www.pearsoned.ca/school/physicssource.

In these sports and many others, putting a spin on the ball is an essential skill. Even though the pitch from a pitcher may be extremely difficult to predict, the behaviour of the ball isn't. If players perform the same motion reliably when they kick, throw, or hit the ball, it will always behave the same way. That is why good players, when faced with the curving soccer ball or fastball, can anticipate where the ball will be and adjust their positions.

It is accurate to say that the physical properties of a spinning ball or anything moving with circular motion can be predicted. In fact, the rotational velocity, frequency, centripetal force, and radius of a spinning object can be related mathematically.

MINDS ON Spinning Objects in Sports

In groups of two or three, think of as many sports as you can that involve a spinning motion. It may be the player or an object that has the spin.

Indicate what advantages the spinning motion has for the player and what type of motion is used to cause the spin. Discuss your answers with the class.

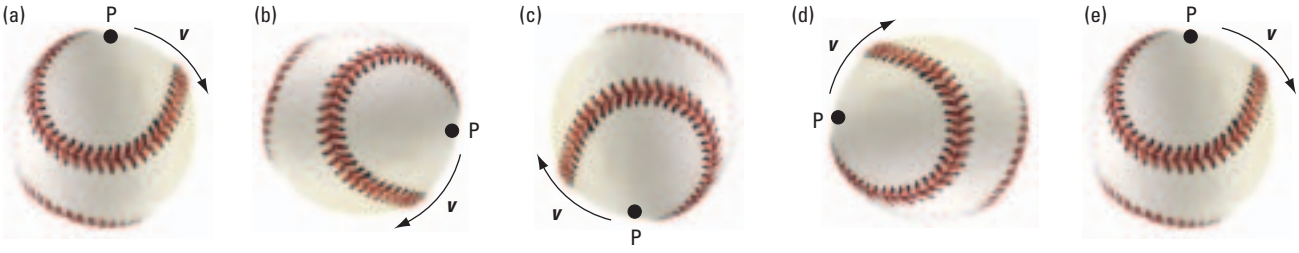
Period and Frequency of Circular Motion

A baseball pitcher can throw a baseball at speeds of about 145 km/h. By flicking his wrist, he can give the ball a spin so that, in effect, it has two velocities: a velocity as it approaches the batter, and a rotational velocity because of its spin (Figure 5.15). The rotational velocity can be measured indirectly by measuring the time it takes for one complete rotation. One complete rotation is called a **cycle** or **revolution**, and the time for one cycle is the **period** (T), measured in s/cycle.

cycle: one complete back-and-forth motion or oscillation

revolution: one complete cycle for an object moving in a circular path

period: the time required for an object to make one complete oscillation (cycle)



▲ **Figure 5.15** Point P on the spinning baseball makes one complete rotation from (a) to (e). The time for this is called the period and is measured in s/cycle. This is frequently abbreviated to s for convenience.

If an object is spinning quickly, the period may be a fraction of a second. For example, a hard drive in a computer makes one complete revolution in about 0.00833 s. This value is hard to grasp and is inconvenient to use. It is often easier to measure the number of rotations in a certain amount of time instead of the period.

Frequency (f) is a measurement that indicates the number of cycles an object makes in a certain amount of time, usually one second. The SI units for frequency are cycles/s or hertz (Hz). You might have noticed that the units for frequency are cycles/s while the units for period are s/cycle. Each is the inverse of the other, so the relationship can be expressed mathematically as:

frequency: the number of cycles per second measured in hertz (Hz)

$$T = \frac{1}{f} \text{ or } f = \frac{1}{T}$$

You may also have seen rotational frequency expressed in **rpm**. Even though this unit for measuring frequency is not an SI unit, it is commonly used commercially in products such as power tools. An rpm is a revolution per minute and is different from a hertz. It represents the number of revolutions in one minute instead of one second, so it is always 60 times bigger than the value in Hz. A simple method can be used to convert Hz to rpm and vice versa:

rpm: revolutions per minute

$$\text{Hz} \underset{\div\ 60\ \text{s/min}}{\overset{\times\ 60\ \text{s/min}}{\rightleftarrows}} \text{rpm}$$

Example 5.1

The hard drive in Figure 5.16 stores data on a thin magnetic platter that spins at high speed. The platter makes one complete revolution in 0.00833 s. Determine its frequency in Hz and rpm.

Given

$T = 0.00833$ s

Required

frequency in Hz and rpm

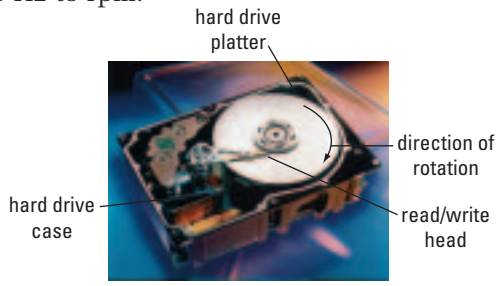
Analysis and Solution

The frequency is the inverse of the period. Solve the frequency of the hard drive in the SI unit for frequency (Hz) and then convert the Hz to rpm.

$$T = \frac{1}{f}$$

$$f = \frac{1}{0.00833 \text{ s}}$$

$$= 120 \text{ Hz}$$



▲ **Figure 5.16**

Now convert the SI units of frequency to rpm:

$$\text{Hz} \underset{\div 60 \text{ s/min}}{\overset{\times 60 \text{ s/min}}{\rightleftarrows}} \text{rpm}$$

$$120 \text{ Hz} \times 60 \frac{\text{s}}{\text{min}} = 7.20 \times 10^3 \text{ rpm}$$

Paraphrase

The frequency of the hard drive is 120 Hz or 7.20×10^3 rpm.

Practice Problems

1. The propeller of a toy airplane rotates at 300 rpm. What is its frequency?
2. An electric motor rotates at a frequency of 40 Hz. What is its rotational frequency in rpm?
3. A medical centrifuge is a device that separates blood into its parts. The centrifuge can spin at up to 6.0×10^4 rpm. What is its frequency in hertz and what is its period?

Answers

1. 5.00 Hz
2. 2.4×10^3 rpm
3. 1.0×10^3 Hz; 1.0×10^{-3} s

Speed and Circular Motion

At the beginning of this chapter you learned that, at this moment, you are moving at approximately 107 000 km/h as Earth moves in its orbit around the Sun. It's hard to imagine that Earth is moving that fast through our solar system. How was the speed determined?

The answer is the simple application of an equation you learned in section 1.2 of chapter 1:

$$v = \frac{\Delta d}{\Delta t}$$

where Δd is the distance travelled and Δt is the time that it takes the object to travel that distance.

In the case of circular motion, the distance around a circle is the circumference (C), given by $C = 2\pi r$. The time it takes for one revolution is the period (T). Therefore, the speed of anything moving with uniform circular motion can be described by the equation:

$$v = \frac{2\pi r}{T} \qquad (1)$$

where r is the radius in metres, T is the period in seconds, and v is the speed in metres per second.

Let's look at Earth as it follows a circular orbit around the Sun. Earth has an orbital radius of approximately 1.49×10^8 km, and makes one complete revolution in 365.25 days. By substituting these values into the equation for speed, we can do the following calculation:

$$\begin{aligned} v &= \frac{2\pi r}{T} \\ &= \frac{2\pi(1.49 \times 10^{11}\ \text{m})}{3.16 \times 10^7\ \text{s}} \\ &= 2.97 \times 10^4\ \text{m/s} \end{aligned}$$

Then convert this to kilometres per hour:

$$2.97 \times 10^4 \frac{\text{m}}{\text{s}} \times \frac{1\ \text{km}}{1000\ \text{m}} \times \frac{3600\ \text{s}}{1\ \text{h}} = 1.07 \times 10^5\ \text{km/h}$$

Earth's speed as it orbits the Sun is approximately 107 000 km/h.

The speed of a planet as it rotates on its axis can be determined in the same way but varies depending on the latitude. We will explore the reasons for this later in "Centripetal Force, Acceleration, and Frequency" in section 5.2.

PHYSICS INSIGHT

Remember: The SI units for distance and time are metres and seconds, respectively.

Example 5.2

A pebble is stuck in the treads of a tire at a distance of 55.0 cm from the axle (Figure 5.17). It takes just 0.40 s for the wheel to make one revolution. What is the speed of the pebble at any instant?

Given

$r = 0.550$ m
$T = 0.40$ s

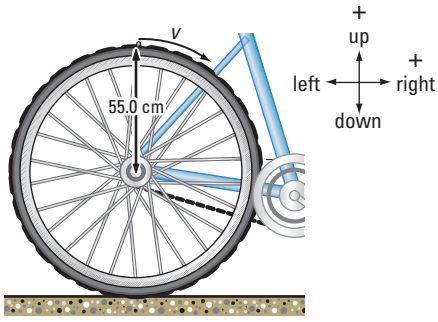
Required

speed (v) of the pebble

Analysis and Solution

Determine the speed by using equation 1:

$$\begin{aligned} v &= \frac{2\pi r}{T} \\ &= \frac{2\pi\,(0.550\ \text{m})}{0.40\ \text{s}} \\ &= 8.6\ \text{m/s} \end{aligned}$$



▲ **Figure 5.17** The speed of the pebble is determined by its distance from the axis of rotation, and the wheel's period (T).

Paraphrase

The speed of the pebble caught in the tire tread is 8.6 m/s.

Practice Problems

1. How much time does it take for the tires of a racecar to make one revolution if the car is travelling at 261.0 km/h and the wheels have a radius of 0.350 m?
2. In 2006, an Alberta astronomer discovered the fastest spinning collapsed star (called a pulsar) ever found. It has a radius of only 16.1 km and is spinning at a rate of 716 Hz (faster than a kitchen blender). What is its speed at its equator?

Answers

1. 0.0303 s
2. 7.24×10^7 m/s

A Closer Look at Centripetal Acceleration and Force

In Unit II, you learned that, just before a race, dragster drivers spin their tires to make them hot and sticky so that the coefficient of friction increases between the tire and the road. When the race starts, the tires will have a better grip and the dragster will be able to accelerate at a greater rate. While the dragster performs the tire-spin, the tires change shape (Figure 5.18). The rear tires start off being fat and thick. During the spin they become thin and their diameter increases. Clearly, this must have something to do with the spinning motion of the wheel, and therefore centripetal force, but what?

▲ **Figure 5.18** (a) At first, the dragster's wheels have a low rotational speed and don't stretch noticeably. The centripetal acceleration and force are small. (b) The dragster's wheels are spinning very fast and stretch away from the rim. The centripetal acceleration and force are large.

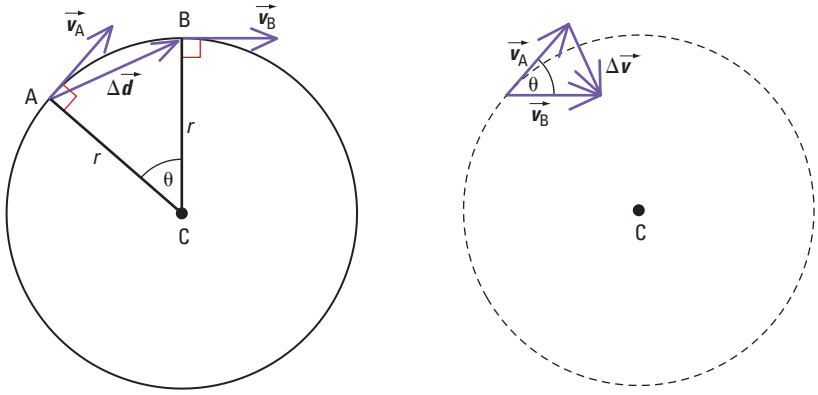
Before the tires start spinning, they are in their natural shape. When they are spinning, they experience a strong centripetal acceleration and force. Both act toward the centre of the wheel. Each tire is fastened to a rim, so the rim is pulling the tire inward. However, the tire moves the way Newton's first law predicts it will — it attempts to keep moving in a straight line in the direction of the velocity. Thus the tire, being made of rubber, stretches. Dragster tires and the tires of trucks and passenger cars are designed to endure high speeds without being torn apart. The faster a wheel spins, the greater the centripetal acceleration and force.

Car tires are thoroughly tested to ensure they can handle a centripetal force much greater than the centripetal force at the speed that you would normally drive in the city or on the highway. However, if you drove at a speed that exceeded the tires' capabilities, the tires could be torn apart.

Is speed the only factor that affects centripetal acceleration and force, or are there other factors? Are the factors that affect centripetal acceleration the same as those that affect the centripetal force? To answer these questions, let's start by taking a closer look at centripetal acceleration.

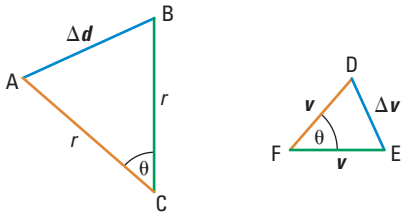
Factors Affecting the Magnitude of Centripetal Acceleration

The rotational speed is one factor that determines the magnitude of the centripetal acceleration, but what other factors play a role? To answer this question, look at Figure 5.19. It shows the two diagrams you saw earlier as Figures 5.9 and 5.10 in section 5.1. Using these figures, we can derive an equation for centripetal acceleration.



▲ **Figure 5.19** An object following in a circular path moves through a displacement $\Delta\vec{d}$ as there is a change in velocity $\Delta\vec{v}$. The triangles formed by these two vectors will help us solve for centripetal acceleration.

As already stated, the two triangles are similar. Therefore, we can compare them. For convenience, the triangles have been redrawn below without the circles (Figure 5.20). Since v_A and v_B have the same value, we have dropped the designations A and B in Figure 5.20. We have omitted the vector arrows because we are solving only for the magnitude of the centripetal acceleration.



▲ **Figure 5.20** Triangles ABC and DEF are similar, so we can use a ratio of similar sides.

Triangle ABC is similar to triangle DEF in Figure 5.20. Therefore, a ratio of similar sides can be created:

$$\frac{\Delta v}{v} = \frac{\Delta d}{r}$$

or

$$\Delta v = \frac{v\Delta d}{r} \qquad (2)$$

Remember, Δv is directed toward the centre of the circle. The time it took for the velocity vectors to move the small distance Δd can be designated Δt. In this time, the Δv vector was created.

Since $v = \frac{\Delta d}{\Delta t}$, we can manipulate the equation so that:

$$\Delta t = \frac{\Delta d}{v} \tag{3}$$

To find acceleration toward the centre of the circle (centripetal acceleration), divide the Δv by Δt.

$$a_c = \frac{\Delta v}{\Delta t} \tag{4}$$

Now we can substitute equations 2 and 3 into equation 4:

$$a_c = \frac{\frac{v\Delta d}{r}}{\frac{\Delta d}{v}}$$

By taking the reciprocal of the denominator and multiplying the two fractions, we have:

$$a_c = \frac{v\cancel{\Delta d}}{r} \times \frac{v}{\cancel{\Delta d}}$$

Simplified, this becomes:

$$a_c = \frac{v^2}{r} \tag{5}$$

where a_c is the centripetal acceleration in metres per second squared toward the centre of the circle, v is the rotational speed of the object moving with uniform circular motion in metres per second, and r is the radius of the circular motion in metres.

The centripetal acceleration depends only on the speed and radius of the circular motion. Mass does not affect it, just as the mass of a falling object does not affect the acceleration of gravity caused by Earth. A truck or a marble will both experience a gravitational acceleration of 9.81 m/s^2 [down]. Two objects of different masses moving in a circular path will experience the same centripetal acceleration if they have the same radius and speed.

Mass does not affect centripetal acceleration, but companies that manufacture racing tires, jet engines, and other equipment know they cannot ignore it. In fact, the mass of a tire or fan blade is important to them. These companies are continually looking for ways to reduce mass without decreasing the strength of their products. Why? The answer has to do with the centripetal force that these devices experience.

If a fan blade or tire has a large mass, it will experience a large centripetal force and might break apart at high speeds. Reducing the mass decreases the centripetal force these parts experience, but often with a trade-off in strength. Next, we will examine the factors that influence centripetal force.

Example 5.3

A DVD disc has a diameter of 12.0 cm and a rotational period of 0.100 s (Figure 5.21). Determine the centripetal acceleration at the outer edge of the disc.

Given

$D = 0.120$ m

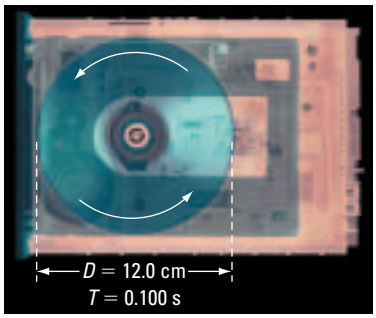
$T = 0.100$ s

Required

centripetal acceleration (a_c)

Analysis and Solution

The magnitude of the centripetal acceleration depends on speed and radius. Convert the diameter to a radius by dividing it by 2.



▲ **Figure 5.21**

$$r = \frac{D}{2}$$

$$= \frac{0.120 \text{ m}}{2}$$

$$= 0.600 \text{ m}$$

Determine the speed of the outer edge of the disc:

$$v = \frac{2\pi r}{T}$$

$$= \frac{2\pi(0.600 \text{ m})}{0.100 \text{ s}}$$

$$= 3.77 \text{ m/s}$$

Now use equation 5 to determine the centripetal acceleration:

$$a_c = \frac{v^2}{r}$$

$$= \frac{\left(3.77 \frac{\text{m}}{\text{s}}\right)^2}{0.0600 \text{ m}}$$

$$= 2.37 \times 10^2 \text{ m/s}^2$$

Note that no vector arrows appear on a_c or v because we are solving for their magnitude only.

Paraphrase

The centripetal acceleration at the edge of the DVD disc is 2.37×10^2 m/s^2.

Practice Problems

1. You throw a Frisbee to your friend. The Frisbee has a diameter of 28.0 cm and makes one turn in 0.110 s. What is the centripetal acceleration at its outer edge?
2. A child playing with a top spins it so that it has a centripetal acceleration of 125.0 m/s^2 at the edge, a distance of 3.00 cm from the axis of rotation. What is the speed at the edge of the top?
3. A helicopter blade has a diameter of 14.0 m and a centripetal acceleration at the tip of 2527.0 m/s^2. What is the period of the helicopter blade?

Answers

1. 4.57×10^2 m/s^2
2. 1.94 m/s
3. 0.331 s

Factors Affecting the Magnitude of Centripetal Force

eTECH

For an interesting interactive simulation of the relationship between velocity and centripetal force, follow the links at www.pearsoned.ca/physicssource.

eMATH

To graph the relationship between centripetal force and speed and determine the mass of the object from the graph, visit www.pearsoned.ca/physicssource.

Mass does not affect centripetal acceleration, but it does influence the force needed to move an object in a circular path. From Newton's second law, we know that the net force is the product of mass and the net acceleration. Therefore, centripetal force must simply be the product of the mass and centripetal acceleration:

$$F_{net} = ma$$

$$F_c = ma_c$$

or

$$F_c = \frac{mv^2}{r} \qquad (6)$$

where m is the mass in kilograms, v is the rotational speed in metres per second of the object moving with uniform circular motion, and r is the radius of the circular motion in metres. Notice that v and r are the same as in equation 5. Therefore, all the factors that affect centripetal acceleration also affect the centripetal force; namely, speed and the radius of rotation. However, the mass affects only the centripetal force.

Example 5.4

Determine the magnitude of the centripetal force exerted by the rim of a dragster's wheel on a 45.0-kg tire. The tire has a 0.480-m radius and is rotating at a speed of 30.0 m/s (Figure 5.22).

Practice Problems

1. An intake fan blade on a jet engine has a mass of 7.50 kg. As it spins, the middle of the blade has a speed of 365.9 m/s and is a distance of 73.7 cm from the axis of rotation. What is the centripetal force on the blade?
2. A 0.0021-kg pebble is stuck in the treads of a dirt bike's wheel. The radius of the wheel is 23.0 cm and the pebble experiences a centripetal force with a magnitude of 0.660 N. What is the speed of the wheel?

Answers

1. 1.36×10^6 N
2. 8.50 m/s

Given

$m = 45.0$ kg
$r = 0.480$ m
$v = 30.0$ m/s

Required

centripetal force exerted on the tire by the rim (F_c)

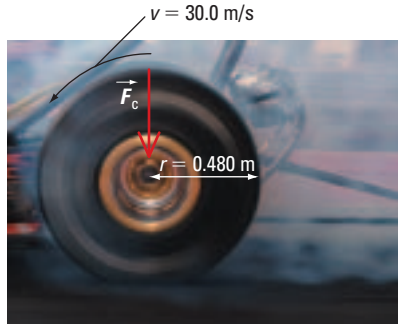
Analysis and Solution

Use equation 6 to solve for the centripetal force.

$$F_c = \frac{mv^2}{r}$$

$$= \frac{(45.0 \text{ kg})\left(30.0 \frac{\text{m}}{\text{s}}\right)^2}{0.480 \text{ m}}$$

$$= 8.44 \times 10^4 \text{ N}$$



▲ **Figure 5.22** The dragster wheel experiences a centripetal force pulling it inward.

Paraphrase

The magnitude of the centripetal force exerted on the tire by the rim is 8.44×10^4 N.

5-3 Inquiry Lab

Required Skills
- Initiating and Planning
- Performing and Recording
- Analyzing and Interpreting
- Communication and Teamwork

Speed and Centripetal Force

Question

What is the relationship between centripetal force and the speed of a mass moving in a horizontal circle?

Hypothesis

State a hypothesis relating centripetal force and speed. Remember to use an "if/then" statement.

Variables

The variables in this lab are the radius of the circular path, mass of the rubber stopper, mass of the hanging mass, number of revolutions, elapsed time, period, and speed. Read the procedure and identify the controlled, manipulated, and responding variables.

Materials and Equipment

1-hole rubber stopper (mass ≦ 25 g)
1.5 m of string or fishing line
0.5-cm-diameter plastic tube
5 masses: 50 g, 100 g, 150 g, 200 g, 250 g
metre-stick
felt marker
safety goggles
stopwatch

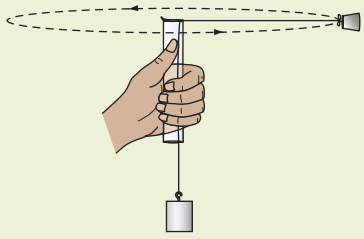
CAUTION: Remember to swing the rubber stopper over your head and in a place clear of obstructions.

Procedure

1. Copy Table 5.3, at the bottom of this page, into your notebook.
2. Secure the rubber stopper to one end of the string.
3. Run the other end of the string through the plastic tube and attach the 50-g mass to it. Record this mass in the "Mass" column of the table. The force of gravity exerted on this mass is the centripetal force.
4. Hold the end of the string attached to the stopper at the zero mark of a metre-stick laid on a table. The zero mark of the ruler should line up with the centre of the stopper. While holding the stopper in position, pull the string taut along the ruler.
5. With the felt marker, mark the string at 40 cm.
6. Hold the rubber stopper in one hand and the plastic tube in the other. Adjust the string's position so that the 40-cm mark is positioned on the lip of the plastic tube. With the hand holding the plastic tube, pinch the string to the lip of the tube using your thumb or forefinger.
7. Put on your safety goggles. Begin spinning the rubber stopper in a horizontal circle above your head as you release the string (Figure 5.23). Make sure the mass is hanging freely. At first, you may have to pull the string up or down using your other hand to position the mark as the stopper is spinning.
8. Adjust the rate at which you spin the rubber stopper so that the string does not slip up or down and the mark stays in position at the lip at the top of the tube. Once you have reached a steady rate, your partner can begin timing. Do not pull the string up or down with the other hand.

▼ **Table 5.3** Data for 5-3 Inquiry Lab

Mass (k)	Time for 20 Revolutions (s)		Average Time (s)	Period (s)	Speed (m/s)	Centripetal Force ($\times 10^{-1}$ N)
	Time 1	Time 2				



▲ **Figure 5.23**

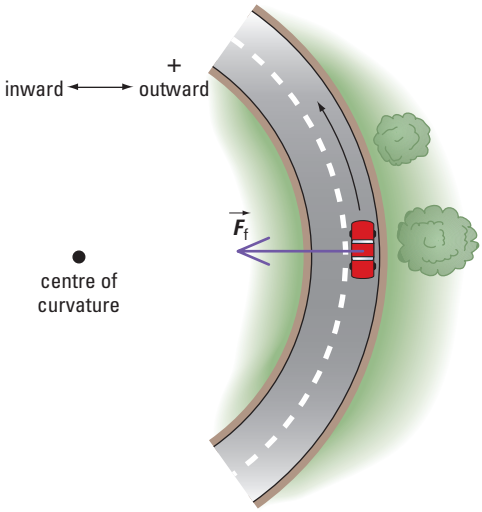
9. Your partner should time 20 complete revolutions of the rubber stopper using a stopwatch. While your partner does this, be sure to monitor the speed of the stopper so that the mark does not move off the lip. Record the time in the "Time 1" column of Table 5.3.

10. Repeat step 9 and record the time in the "Time 2" column of Table 5.3.

11. Increase the hanging mass by 50 g, and record this mass in the "Mass" column of the table. Repeat steps 6 to 10 until all the masses are used.

Analysis

1. For each trial, average the two times, and place the result in the "Average Time" column of the table.
2. For each trial, divide the average time by the number of revolutions the stopper made. Record the value in the "Period" column of the table.
3. For each trial, determine the speed of the stopper using the equation $v = \frac{2\pi r}{T}$. Record the value in the "Speed" column of the table.
4. For each trial determine the force of gravity acting on the mass hanging from the string using the equation $F_g = mg$. Record these values in the "Centripetal Force" column of the table.
5. Identify the controlled, responding, and manipulated variables.
6. Plot a graph of speed versus centripetal force. Remember to plot the manipulated variable on the horizontal axis and the responding variable on the vertical axis.
7. Complete the statement, "The speed varies with …."
8. Was your hypothesis accurate? If not, how can it be modified to reflect your observations in this lab?



▲ **Figure 5.24** A car turning left. The force of friction of the tires on the road is the centripetal force.

A Horizontal System in Circular Motion

Imagine that a car is following a curve to the left on a flat road (Figure 5.24). As the car makes the turn its speed remains constant, and it experiences a centripetal force. The centripetal force is caused by the wheels turning to the left, continually changing the direction of the car. If it weren't for the frictional force between the tire and the road, you would not be able to make a turn. Hence, the frictional force between the tires and the road is the centripetal force. For simplicity, this is written as:

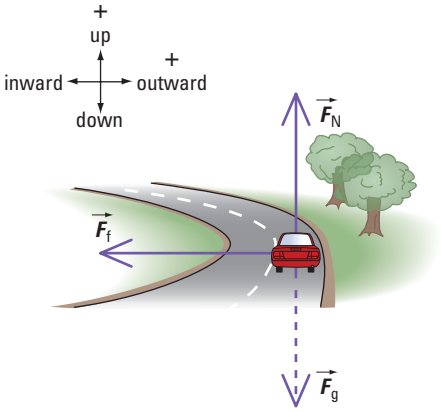
$$F_c = F_f$$

Recall that the magnitude of the force of friction is represented by the equation $F_f = \mu F_N$, where F_N is the normal force, or perpendicular force exerted by the surface on the object. It is equal and opposite to the force of gravity on a horizontal surface. Recall from Unit II that the coefficient of friction, μ, is the magnitude of the force of friction divided by the magnitude of the normal force. Think of it as a measure of how well two surfaces slide over each other. The lower the value, the easier the surfaces move over one another.

Assume that the driver increases the speed as the car turns the corner. As a result, the centripetal force also increases (Figure 5.25). Suppose the force of friction cannot hold the tires to the road. In other words, the force of friction cannot exert the needed centripetal force because of the increase in speed. In that case, the car skids off the road tangentially. Recall that kinetic friction is present when a car slides with its wheels locked. If a car turns a corner without skidding, static friction is present.

PHYSICS INSIGHT

On a horizontal surface the force of gravity (F_g) is equal and opposite to the normal force (F_N).



◀ Figure 5.25 $\vec{F}_c = \vec{F}_f$ The maximum frictional force that can be exerted between the road and the tires determines the maximum speed the car can go around the turn without skidding. For horizontal surfaces, the normal force is equal and opposite to the force of gravity ($\vec{F}_N = -\vec{F}_g$).

Example 5.5

Determine the maximum speed at which a 1500.0-kg car can round a curve that has the radius of 40.0 m, if the coefficient of friction between the tires and the road is 0.60.

Given

$m = 1500.0 \text{ kg}$

$\mu_s = 0.60$

$r = 40.0 \text{ m}$

$g = 9.81 \text{ m/s}^2$

Required

maximum speed (v)

Analysis and Solution

First draw a free-body diagram to show the direction of the forces (Figure 5.26). The normal force is equal to the force of gravity on a horizontal surface.

$$F_c = F_f$$

$$\frac{mv^2}{r} = \mu F_N$$

$$F_N = mg$$

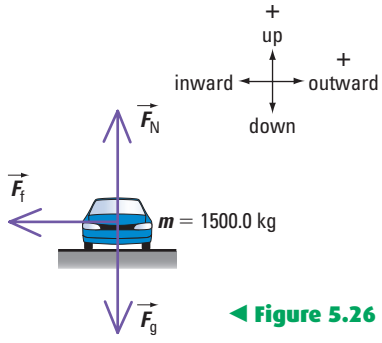
$$\frac{mv^2}{r} = \mu(mg)$$

$$v^2 = \mu g r$$

$$v = \sqrt{\mu g r}$$

$$= \sqrt{(0.60)\left(9.81 \frac{\text{m}}{\text{s}^2}\right)(40.0 \text{ m})}$$

$$= 15.3 \text{ m/s}$$



◀ Figure 5.26

Paraphrase

The fastest that the car can round the curve is 15.3 m/s or 55.2 km/h. If it attempts to go faster, the force of static friction will be insufficient to prevent skidding.

Practice Problems

1. An Edmonton Oiler ($m = 100$ kg) carves a turn with a radius of 7.17 m while skating and feels his skates begin to slip on the ice. What is his speed if the coefficient of static friction between the skates and the ice is 0.80?
2. Automotive manufacturers test the handling ability of a new car design by driving a prototype on a test track in a large circle ($r = 100$ m) at ever-increasing speeds until the car begins to skid. A prototype car ($m = 1200$ kg) is tested and found to skid at a speed of 95.0 km/h. What is the coefficient of static friction between the car tires and the track?
3. A 600.0-g toy radio-controlled car can make a turn at a speed of 3.0 m/s on the kitchen floor where the coefficient of static friction is 0.90. What is the radius of its turn?

Answers

1. 7.5 m/s
2. 0.710
3. 1.0 m

▲ **Figure 5.27** Why doesn't this car fall off the track at the top of the loop?

Centripetal Force and Gravity

The designers of amusement park rides know their physics. Rides will toss you around, but leave you unharmed. Many of these rides spin you in circles — the Ferris wheel and the roller coaster, for example. The roller coaster often has a vertical loop somewhere along its track (Figure 5.27). Why is it that when you reach the top of the loop, where the car is inverted, you don't fall out? It isn't because of the harness that they put over you before you start the ride. That just keeps you strapped into the car so you don't do something silly like stand up while the roller coaster is moving. No, the answer lies in the physics of the roller coaster's design.

A Vertical System in Circular Motion

All roller coasters, regardless of their appearance, are designed so that each car has sufficient velocity to remain in contact with the track at the top of the loop. At the top, the centripetal force is exerted by two forces working in the same direction: the track on the car, which is the normal force ($\vec{F}_N$) and the force of gravity ($\vec{F}_g$). Both forces push the car toward the centre of the loop.

The speed of the car determines the amount of centripetal force needed to maintain a certain radius. As you saw from equation 6, the centripetal force is directly related to the square of the speed. Here is equation 6 again:

$$F_c = \frac{mv^2}{r} \qquad (6)$$

But the force of gravity is independent of speed, and will always pull the car downward with the same force. To demonstrate the role that gravity plays as a portion of the centripetal force, we can look at a hypothetical situation of a roller coaster going around a loop as shown in Figures 5.28(a), (b), and (c).

Assume a roller coaster car like the one in Figure 5.28 on the opposite page experiences a force of gravity that is 1000 N. This value won't change regardless of the car's position on the track or its speed. Remember, the centripetal force is the net force. In this case, it is equal to the sum of the gravitational force ($\vec{F}_g$) and the track's force ($\vec{F}_N$) on the car.

Figure 5.28 illustrates how speed affects the roller coaster car's motion when it is sent through the loop three times. Each time, it is sent with less speed.

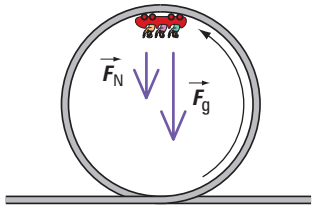
info BIT

Most roller coaster loops use a shape called a clothoid, where the curvature increases at the top. This reduces the speed needed to move safely through the loop. It also adds thrills by having relatively longer vertical parts in the loop.

PHYSICS INSIGHT

$\vec{F}_c$ = centripetal force
$\vec{F}_g$ = force due to gravity
$\vec{F}_N$ = force that the track exerts on the car

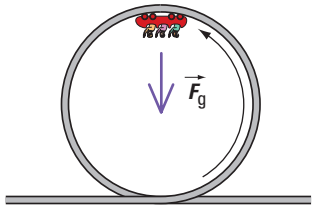
▶ **Figure 5.28(a)** The first time through the loop, the speed is such that the roller coaster requires a centripetal force of 1500 N to keep it moving in a circular path. At the top of the loop, the roller coaster car will experience a centripetal force that is the sum of the force of gravity and the force exerted by the track, pushing the car inward to the centre of the circle. The centripetal force acts down, so it is −1500 N. The force of gravity is constant at 1000 N so the track pushes inward with 500 N to produce the required centripetal force. The car goes around the loop with no problem.



$\vec{F}_c = \vec{F}_g + \vec{F}_N$

$-1500\ \text{N} = -1000\ \text{N} + -500\ \text{N}$

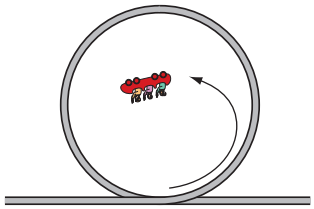
▶ **Figure 5.28(b)** Suppose the next time the car goes around the track, it is moving more slowly, so that the centripetal force required is only 1000 N. In this case, the force of gravity alone can provide the required centripetal force. Therefore, the track does not need to exert any force on the car to keep it moving on the track. There is no normal force, so the force of gravity alone is the centripetal force. The car goes around the loop again with no problem.



$\vec{F}_c = \vec{F}_g + \vec{F}_N$

$-1000\ \text{N} = -1000\ \text{N} + 0\ \text{N}$

▶ **Figure 5.28(c)** Now suppose the last time the car goes around the track, it is moving very slowly. The required centripetal force is just 800 N, but the force of gravity is constant, so it is still 1000 N; that is, 200 N more than the centripetal force required to keep the car moving in a circular path with this radius. If the track could somehow pull upward by 200 N to balance the force of gravity, the car would stay on the track. This is something it can't do in our hypothetical case. Since the gravitational force cannot be balanced by the track's force, it pulls the car downward off the track.



$\vec{F}_c = \vec{F}_g + \vec{F}_N$

$-800\ \text{N} = -1000\ \text{N} + 200\ \text{N}$

Since the normal force cannot pull upward, it cannot generate +200 N. 200 N more is needed to keep the car on a track of this radius, so the car falls off.

The slowest that any car can go around the track would be at a speed that requires a centripetal force that has a magnitude equal to gravity. Gravity would make up all of the centripetal force. This can be expressed as:

$$F_c = F_g$$

Remember that this equality doesn't mean that the centripetal force is a different force than the force of gravity. It means that it *is* the force of gravity.

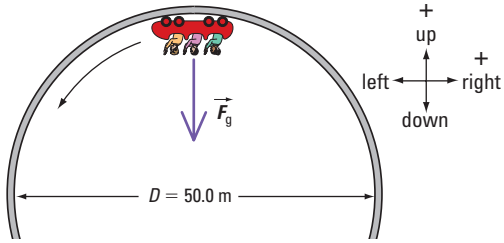
All roller coasters are designed so that the cars' speed is enough to create a centripetal force greater than the force of gravity to minimize the chance of the car leaving the track. The wheels of roller coaster cars also wrap around both sides of the track so the track can indeed pull upwards.

Concept Check

The centripetal force exerted on the Moon as it orbits Earth is caused by Earth's gravity. What would happen to the Moon's orbit if the Moon's velocity increased or slowed down?

Example 5.6

A 700.0-kg roller coaster car full of people goes around a vertical loop that has a diameter of 50.0 m (Figure 5.29). What minimum speed must the roller coaster car have at the top of the vertical loop to stay on the track?



▲ **Figure 5.29** When the roller coaster is moving with the minimum speed to maintain its circular path, the force of gravity alone is the centripetal force. The track exerts no force on the car.

Analysis and Solution

For the roller coaster to stay on the track at the top of the loop with the minimum speed, the centripetal force is the force of gravity.

To determine the radius of the loop, divide the diameter by 2.

$$r = \frac{50.0\text{ m}}{2}$$
$$= 25.0\text{ m}$$

Use the equality of the centripetal force and gravity to solve for the speed:

$$F_{net} = F_g$$
$$F_c = F_g$$
$$\frac{mv^2}{r} = mg$$
$$\frac{v^2}{r} = g$$
$$v = \sqrt{rg}$$
$$= \sqrt{25.0\text{ m}\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}$$
$$= 15.7\text{ m/s}$$

The roller coaster car must have a minimum speed of 15.7 m/s to stay on the track.

Practice Problems

1. Neglecting friction, what is the minimum speed a toy car must have to go around a vertical loop of diameter 15.0 cm without falling off?
2. What is the maximum radius a roller coaster loop can be if a car with a speed of 20.0 m/s is to go around safely?

Answers

1. 1.21 m/s
2. 40.8 m

e LAB

For a probeware activity that investigates circular motion in a vertical plane, follow the links at www.pearsoned.ca/physicssource.

PHYSICS INSIGHT

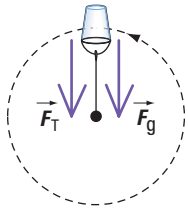
All objects fall at a rate of 9.81 m/s^2 regardless of their mass.

You can swing a full pail of water around in a circle over your head without getting wet for the same reason that a roller coaster can go around a loop without falling off the track. Let's examine the case of a mass on the end of a rope, moving in a vertical circle, and see how it compares to the roller coaster.

A bucket of water is tied to the end of a rope and spun in a vertical circle. It has sufficient velocity to keep it moving in a circular path. Figures 5.30(a), (b), and (c) show the bucket in three different positions as it moves around in a vertical circle.

▶ **Figure 5.30(a)** The bucket is at the top of the circle. In this position, two forces are acting on the bucket: the force of gravity and the tension of the rope. Both are producing the centripetal force and are acting downward. The equation to represent this situation is:
$\vec{F}_c = \vec{F}_g + \vec{F}_T$

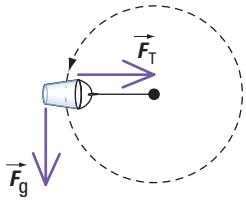
(a)



$\vec{F}_c = \vec{F}_g + \vec{F}_T$

▶ **Figure 5.30(b)** When the bucket has moved to the position where the rope is parallel to the ground, the force of gravity is perpendicular to the tension. It does not contribute to the centripetal force. The tension alone is the centripetal force. We can write this mathematically as:
$\vec{F}_c = \vec{F}_T$

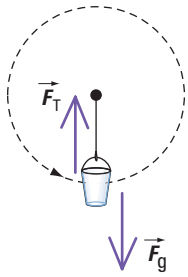
(b)



$\vec{F}_c = \vec{F}_T$

▶ **Figure 5.30(c)** As the bucket moves through the bottom of the circle, it must have a centripetal force that overcomes gravity. The tension is the greatest here because gravity is acting opposite to the centripetal force. The equation is the same as in (a) above, but tension is acting upward, so when the values are placed into the equation this time, $\vec{F}_T$ is positive and $\vec{F}_g$ is negative. The effect is demonstrated in Example 5.7.

(c)



$\vec{F}_c = \vec{F}_g + \vec{F}_T$

Concept Check

1. A bucket filled with sand swings in a vertical circle at the end of a rope with increasing speed. At some moment, the tension on the rope will exceed the rope's strength, and the rope will break. In what position in the bucket's circular path is this most likely to happen? Explain.
2. Is it necessary to know the position of an object moving in a vertical circle with uniform speed if you are determining centripetal force? Explain.

eSIM

To learn more about centripetal force for vertical circular motion, visit www.pearsoned.ca/physicssource.

Forces Affecting an Object Moving in a Vertical Circle

In summary, an object moving in a vertical circle is affected by the following forces:

- The centripetal force is the net force on the object in any position.
- The centripetal force is determined by the object's mass, speed, and radius. In the case of the roller coaster and bucket of water, their mass and radius of curvature are constant so only their speed affects the centripetal force.
- The force of gravity is one of the forces that contributes to the centripetal force.
- The force of gravity remains constant regardless of the position of the object.

Example 5.7

A bucket of water with a mass of 1.5 kg is spun in a vertical circle on a rope. The radius of the circle is 0.75 m and the speed of the bucket is 3.00 m/s. What is the tension on the rope in position C, as shown in Figure 5.31?

Practice Problems

1. Using the information in Example 5.7, determine the tension in the rope at position A in Figure 5.31.
2. Using the information in Example 5.7, determine the tension in the rope in position B in Figure 5.31.
3. A 0.98-kg rock is attached to a 0.40-m rope and spun in a vertical circle. The tension on the rope when the rock is at the top of the swing is 79.0 N [down]. What is the speed of the rock?

Answers

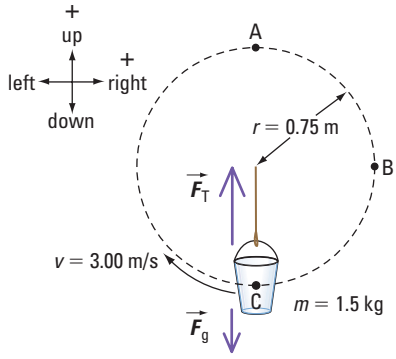
1. 3.3 N [down]
2. 18 N [left]
3. 6.0 m/s

Given

$r = 0.75$ m
$m = 1.5$ kg
$v = 3.00$ m/s
$g = 9.81$ m/s^2

Required

tension ($\vec{F}_T$)



▲ **Figure 5.31**

Analysis and Solution

The centripetal force acting on the bucket will not change as the bucket moves in a vertical circle. The tension will change as the bucket moves in its circular path because gravity will work with it at the top and against it at the bottom.

In position C, the force of gravity works downward (negative), but the centripetal force and tension act upward (positive). Tension must overcome gravity to provide the centripetal force (Figure 5.32).

Remember that the centripetal force is the net force and is the vector sum of all the forces acting on the bucket. Therefore, the equation is:

$$\vec{F}_{net} = \vec{F}_g + \vec{F}_T$$

$$\vec{F}_c = \vec{F}_g + \vec{F}_T$$

$$F_c = F_g + F_T$$

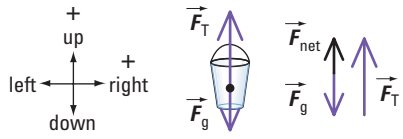
$$F_T = F_c - F_g$$

$$= \frac{mv^2}{r} - (-mg)$$

$$= \frac{(1.5 \text{ kg})\left(3.0 \frac{\text{m}}{\text{s}}\right)^2}{0.75 \text{ m}} - \left[-(1.5 \text{ kg})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)\right]$$

$$= 18\text{N} - [-14.715 \text{ N}]$$

$$= 33 \text{ N}$$



▲ **Figure 5.32**

Paraphrase

The tension on the rope at position C is 33 N [up]. This is the maximum tension the system will experience because gravity acts in the opposite direction of the centripetal force.

Centripetal Force, Acceleration, and Frequency

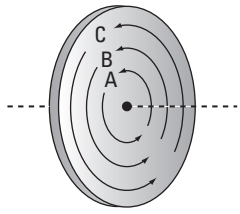
At your local hardware store, you will find a variety of rotary power tools that operate by circular motion. The table saw, circular saw, impact wrench, reciprocating saw, and rotary hammer are a few (Figure 5.33). One of the selling features listed on the box of most of these tools is the rotational frequency (in rpm). At the beginning of this chapter, you learned that rpm refers to the frequency of rotation measured in revolutions per minute. This is an Imperial measurement that has been around for hundreds of years. It has probably persisted because people have a "feel" for what it means.

Even though revolutions per minute (rpm) is not, strictly speaking, the correct way to represent the frequency of rotation, it is a very useful measurement nevertheless. Why is the frequency of rotation often a more useful measure than the speed? The answer has to do with the nature of a rotating object.

▲ **Figure 5.33** Most tools, motors, and devices that rotate with high speed report the frequency of rotation using rpm instead of the speed.

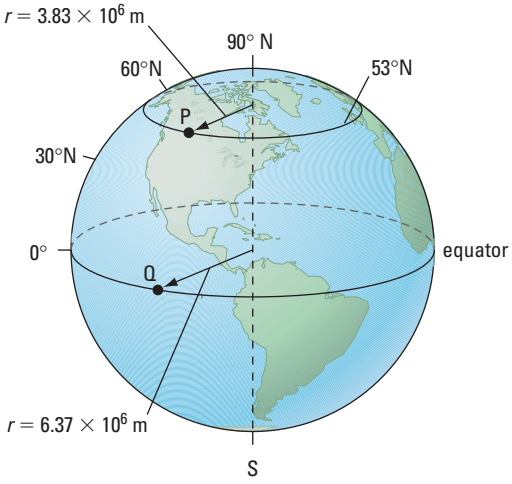
The Effect of Radius on Speed, Period, and Frequency

Imagine a disc spinning about its axis with uniform circular motion (Figure 5.34). Positions A, B, and C are at different radii from the axis of rotation, but all the positions make one complete revolution in exactly the same time, so they have the same period. Of course, if the periods for points A, B, and C are the same, so are their frequencies, and we can make the following generalization: For any solid rotating object, regardless of its shape, the frequency of rotation for all points on the object will be the same.



▲ **Figure 5.34** The velocities at A, B, and C are all different, whereas the rotational frequency of this disc is the same at any point.

Compare the speeds of points A, B, and C. Point A is the closest to the axis of rotation and has the smallest radius, followed by B, and then C with the largest radius. As already discussed, all three points make one complete revolution in the same amount of time, so point C, which has the farthest distance to cover, moves the fastest. Point B moves more slowly than C because it has less distance to travel. Point A has the slowest speed because it has the least distance to cover in the same amount of time. In essence, the speed of the spinning disc changes depending on which point you are referring to. In other words, the speed of a point on a disc changes with respect to its radius.



▲ **Figure 5.35** The speed of any point on Earth depends on its latitude (which determines its rotational radius). Point P moves more slowly than point Q, but they both have the same period and frequency.

At the beginning of this chapter, you learned that the speed of an Albertan is roughly 1000 km/h as Earth rotates on its axis. However, not every point on Earth's surface moves with the same speed. Remember: Speed changes with radius, and on Earth, the distance from the axis of rotation changes with latitude (Figure 5.35). The fastest motion is at the equator and the slowest is at the poles, but every point on Earth has the same *period* of rotation — one day.

Determining Centripetal Force Using Period and Frequency

In earlier example problems, such as Example 5.3, a given rotational frequency or period had to be converted to speed before the centripetal acceleration or force could be determined. It would be simpler to derive the equations for centripetal acceleration and force using rotational frequency or period to save a step in our calculations. The equations for centripetal acceleration and force are:

$$a_c = \frac{v^2}{r} \text{ and } F_c = \frac{mv^2}{r}$$

Recall that:

$$v = \frac{2\pi r}{T}$$

By substituting the velocity equation into the centripetal acceleration equation, the result is:

$$a_c = \frac{(2\pi r)^2}{rT^2}$$

$$= \frac{4\pi^2 r^2}{rT^2}$$

$$a_c = \frac{4\pi^2 r}{T^2} \quad (7)$$

The centripetal force is $F_c = ma$, so the centripetal acceleration is:

$$F_c = \frac{4\pi^2 mr}{T^2} \quad (8)$$

Period is just the inverse of frequency, so it is relatively simple to express equations 7 and 8 in terms of frequency:

$$a_c = 4\pi^2 r f^2 \quad (9)$$

and

$$F_c = 4\pi^2 mr f^2 \quad (10)$$

To convert rpm to hertz, simply divide by 60.

Example 5.8

The compressor blades in a jet engine have a diameter of 42.0 cm and turn at 15 960 rpm (Figure 5.36). Determine the magnitude of the centripetal acceleration at the tip of each compressor blade.

▲ **Figure 5.36** The centripetal acceleration at the tip of a blade can be determined from the frequency of the blade's rotation.

Analysis and Solution

First convert the frequency to SI units (Hz) and determine the radius. Then use equation 9.

$$f = \frac{15\ 960\ \text{rev}}{1\ \text{min}} \times \frac{1\ \text{min}}{60\ \text{s}}$$

$$= 266.00\ \text{Hz}$$

$$r = \frac{D}{2}$$

$$= \frac{0.420\ \text{m}}{2}$$

$$= 0.210\ \text{m}$$

$$a_c = 4\pi^2 r f^2$$

$$= 4\pi^2(0.210\ \text{m})(266.00\ \text{Hz})^2$$

$$= 5.87 \times 10^5\ \text{m/s}^2$$

The magnitude of the centripetal acceleration at the tip of each compressor blade is $5.87 \times 10^5\ \text{m/s}^2$.

Practice Problems

1. A space station shaped like a wheel could be used to create artificial gravity for astronauts living in space. The astronauts would work on the rim of the station as it spins. If the radius of the space station is 30.0 m, what would its frequency have to be to simulate the gravity of Earth ($g = 9.81\ \text{m/s}^2$)?
2. A 454.0-g mass, attached to the end of a 1.50-m rope, is swung in a horizontal circle with a frequency of 150.0 rpm. Determine the centripetal force acting on the mass.

Answers

1. 9.10×10^{-2} Hz
2. 1.68×10^2 N

5.2 Check and Reflect

Knowledge

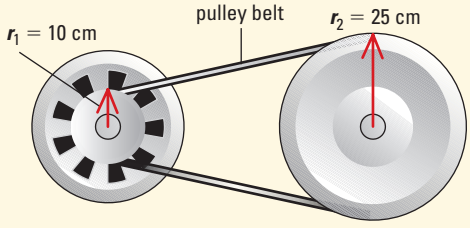
1. A car heading north begins to make a right turn. One-quarter of a whole turn later, in what direction is the centripetal force acting?
2. Does the Moon experience centripetal acceleration? Why?
3. What two things could you do to increase the centripetal force acting on an object moving in a horizontal circle?
4. An object moves in a vertical circle at the end of a rope. As the object moves, explain what happens to:
   (a) the force of gravity, and
   (b) the tension.
5. What force acts as the centripetal force for a plane that is making a horizontal turn?

Applications

6. A car's wheels have a radius of 0.5 m. If the car travels at a speed of 15.0 m/s, what is the period of rotation of the wheels?
7. A propeller blade has a period of rotation of 0.0400 s. What is the velocity of the outer tip of the propeller blade if the tip is 1.20 m from the hub?
8. A 1500-kg car is making a turn with a 100.0-m radius on a road where the coefficient of static friction is 0.70. What is the maximum speed the car can go without skidding?
9. A car rounds a curve of radius 90.0 m at a speed of 100.0 km/h. Determine the centripetal acceleration of the car.
10. NASA uses a centrifuge that spins astronauts around in a capsule at the end of an 8.9-m metallic arm. The centrifuge spins at 35 rpm. Determine the centripetal acceleration that the astronauts feel. How many times greater is this acceleration than the acceleration of gravity?
11. What minimum speed must a toy racecar have to move successfully through a vertical loop that has a diameter of 30.0 cm?
12. Determine the centripetal acceleration acting on a person standing at the equator ($r_{\text{Earth}} = 6.37 \times 10^6$ m).
13. An ant climbs onto the side of a bicycle tire a distance of 0.40 m from the hub. If the 0.010-g ant can hold onto the tire with the force of 4.34×10^{-4} N, at what frequency would the tire fling the ant off?

Extensions

14. Two pulleys are connected together by a belt as shown in the diagram below. If the pulley connected to the motor spins at 200.0 rpm, what is the frequency of the larger pulley? (Hint: Both pulleys have the same velocity at their outer edge.)



15. Two NASCAR racecars go into a turn beside each other. If they remain side by side in the turn, which car has the advantage coming out of the turn?

eTEST

To check your understanding of circular motion and Newton's laws, follow the eTest links at www.pearsoned.ca/school/physicssource.

5.3 Satellites and Celestial Bodies in Circular Motion

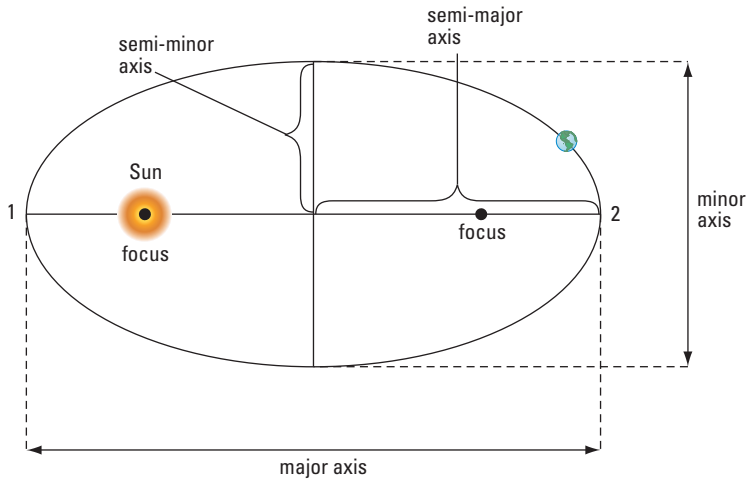
Johannes Kepler (1571–1630) was a German mathematician with a strong interest in astronomy. When he started working for renowned Danish astronomer Tycho Brahe (1546–1601) in 1600, he was given the problem of determining why the orbit of Mars didn't completely agree with mathematical predictions. Brahe probably gave Kepler this job to keep him busy since he didn't share Kepler's ideas that the planets revolved around the Sun. At this time, most people, including scientists, believed that the planets and the Sun revolved around Earth.

info BIT

Comets are objects in space that have very elliptical orbits. They are often difficult to detect because they are relatively small and their orbits take them a great distance from the Sun. It is likely there are many comets orbiting the Sun that we have yet to discover.

Kepler's Laws

Brahe had made very meticulous observations of the orbital position of Mars. Kepler used this data to determine that the planet must revolve around the Sun. Kepler also hypothesized that Mars had an elliptical orbit, not a circular one. Until this time, all mathematical predictions of a planet's position in space were based on the assumption that it moved in a circular orbit. That is why Brahe's observations disagreed with mathematical predictions at the time. By recognizing that planets move in elliptical orbits, Kepler could account for the discrepancy.



▲ **Figure 5.37** Kepler described the motion of planets as an ellipse with a semi-major axis and a semi-minor axis. The average orbital radius is the semi-major axis.

Kepler's First Law

An ellipse is an elongated circle. Figure 5.37 is an example of an ellipse. There are two foci, as well as major and minor axes. In Kepler's model, the Sun is at one focus and the planet's orbit is the path described by the shape of the ellipse. It is clear from Figure 5.37 that the planet will be closer to the Sun in position 1 than in position 2. This means that the planet's orbital radius must be changing as time goes by.

Why then is a planet's orbital radius often written as a fixed number? There are two reasons:

- The radius used is an average value. Mathematically, this average radius can be shown to be the same length as the semi-major axis.
- The orbit of the planet, although an ellipse, is very close to being a circle, so the orbital radius really doesn't change much from position 1 to position 2.

Figure 5.37 is an exaggeration of a planet's orbit for the purposes of clarity.

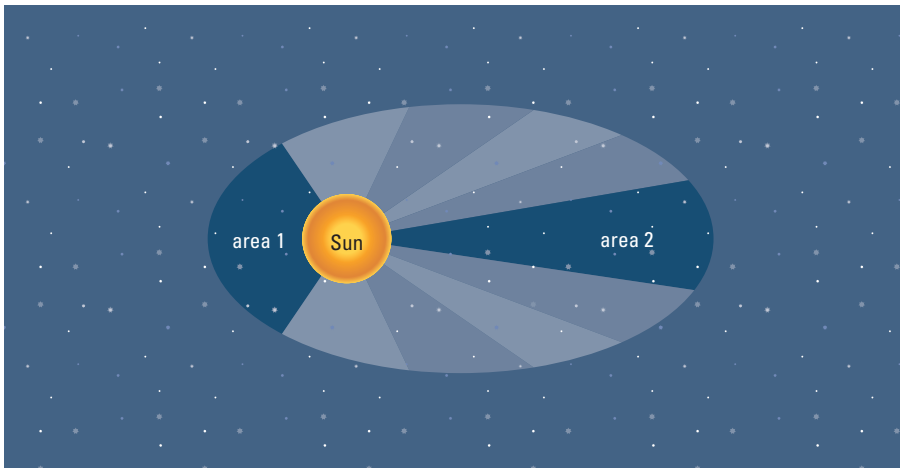
The degree to which an ellipse is elongated is called the eccentricity. It is a number between 0 and 1, with 0 being a perfect circle and 1 being a parabola. Table 5.4 shows the eccentricities of the orbits of the planets and other celestial bodies in our solar system.

▼ **Table 5.4** Eccentricities of Orbits of Celestial Bodies

Celestial Body	Eccentricity
Mercury	0.205
Venus	0.007
Earth	0.017
Mars	0.093
Ceres	0.080
Jupiter	0.048
Saturn	0.054
Uranus	0.047
Neptune	0.009
Pluto	0.249
Eris	0.437
Sedna	0.857

Kepler's Second Law

When Kepler stated his theory, the time was right and the scientific climate was such that it wasn't rejected. He went on to state two more revolutionary ideas relating to the orbits of planets in the solar system. His second law stated that planets move through their elliptical orbit in such a manner as to sweep out equal areas in equal times. Imagine a line that extends from the Sun to the planet. As the planet moves in its orbit, the line moves with it and sweeps out an area. In the same amount of time, at any other position in the planet's orbit, the planet will again sweep out the same area (Figure 5.38).

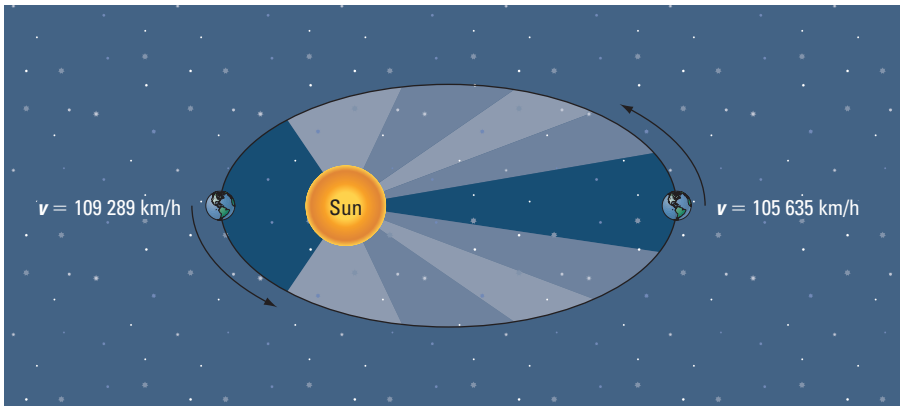


▲ **Figure 5.38** Kepler's second law states that a line drawn from the Sun to the planet sweeps out equal areas in equal times.

eWEB

Sedna was discovered in 2004 and is the largest object yet found in a region of space known as the Oort cloud. On July 29, 2005, an object now named Eris was confirmed. It is larger than Pluto but also in the Kuiper belt. To learn more about these celestial bodies, the Kuiper belt, and the Oort cloud, follow the links at:www.pearsoned.ca/school/physicssource.

One consequence of this rule is that a planet's speed must change throughout its orbit. Consider Figure 5.39 where area 1 is equal to area 2. As the planet approaches the Sun, the orbital radius decreases. If it is to sweep out an area equal to area 2, the planet must speed up and cover a larger distance to compensate for the smaller radius. As the planet gets farther from the Sun, the orbital radius gets larger. The planet slows down and sweeps out the same area again in the same amount of time (Figure 5.39).



▲ **Figure 5.39** Earth's speed varies from 105 635 km/h at its farthest distance from the Sun to 109 289 km/h when it is closest. Its speed changes to compensate for the change in its orbital radius.

Kepler's Third Law

Kepler's third law states that the ratio of a planet's orbital period squared divided by its orbital radius cubed is a constant that is the same for all the planets orbiting the Sun. Written mathematically, it is:

$$\frac{T_a^2}{r_a^3} = K \qquad (11)$$

where T_a is the orbital period of planet A, r_a is the orbital radius of planet A, and K is Kepler's constant.

Since the constant K applies to all planets orbiting the Sun, we can equate the ratio T^2/r^3 between any two planets and write the equation:

$$\frac{T_a^2}{r_a^3} = \frac{T_b^2}{r_b^3} \qquad (12)$$

Until Kepler noticed this relationship between the planets, there was really no indication that the third law would be true. If it hadn't been for Tycho Brahe's accurate measurements of the planets' orbital positions throughout their year, it is unlikely that Kepler would have made this discovery. Once the relationship was known, it was easy to verify, and it further bolstered the credibility of the heliocentric (Sun-centred) model of the solar system. It is important to note that when Kepler derived his third law, he applied it only to planets orbiting the Sun. However, this law can be extended to moons that orbit a planet. In fact, in the most general sense, Kepler's third law is applicable to all celestial bodies that orbit the same focus. For bodies orbiting a different focus, *Kepler's constant will be different*.

Kepler's three laws can be summarized this way:

1. All planets in the solar system have elliptical orbits with the Sun at one focus.
2. A line drawn from the Sun to a planet sweeps out equal areas in equal times.
3. The ratio of a planet's orbital period squared to its orbital radius cubed is a constant. All objects orbiting the same focus (e.g., planets, the Sun) have the same constant. $\frac{T_a^2}{r_a^3} = \frac{T_b^2}{r_b^3}$

eSIM

To learn more about Kepler's second law regarding eccentricity, orbital period, and speed of a planet, visit www.pearsoned.ca/school/physicssource.

eWEB

The Titius-Bode law is another mathematical description that predicts the orbital radius of the planets. To learn more about the Titius-Bode law, follow the links at www.pearsoned.ca/school/physicssource.

Determining Kepler's Constant

Kepler's third law states that the constant K is the same for all planets in a solar system. The period and orbital radius of Earth are well known, so they are used to compute the constant. The mean (average) orbital distance for Earth from the Sun is 1.49×10^{11} m, and Earth's orbital period is 31 557 600 s. Kepler's constant can be calculated as shown below:

$$K = \frac{T_E^2}{r_E^3}$$

$$= \frac{(3.15576 \times 10^7 \text{ s})^2}{(1.49 \times 10^{11} \text{ m})^3}$$

$$= 3.01 \times 10^{-19} \text{ s}^2/\text{m}^3$$

PHYSICS INSIGHT

The orbital radius is always measured from the centre of the orbiting body to the centre of the body being orbited.

info BIT

Some scientists speculate that Pluto and Charon might have been objects from the Kuiper belt that were attracted into an orbit of the Sun by Neptune because of their small size and large orbital radius. The Kuiper belt is a large band of rocky debris that lies 30 to 50 AU from the Sun.

e MATH

Kepler's constant applies only to bodies orbiting the same focus. To explore this concept in more depth and to determine Kepler's constant for Jupiter's moons, visit www.pearsoned.ca/physicssource.

However, you will not often find Kepler's constant written with this value. The reason is twofold. If different units for distance and time are used (something other than metres and seconds), then the constant can be made to equal 1. This has obvious mathematical benefits. The other reason is that using metres and seconds as the units of measurement is impractical when dealing with the scale of our solar system. To represent astronomical distances, it becomes necessary to use units bigger than a metre or even a kilometre. For example, measuring the distance from Earth to the Sun in kilometres (149 000 000 km) is roughly like measuring the distance from Edmonton to Red Deer in millimetres.

Astronomical Units

A more suitable measurement for astronomical distances has been adopted. It is called the astronomical unit (AU). One astronomical unit is the mean orbital distance from Earth to the Sun (the length of the semi-major axis of Earth's orbit). This is a more manageable unit to use. For example, Neptune is only 30.1 AU away from the Sun on average.

Kepler's constant, using units of years and AU, can be determined as follows:

$$K = \frac{T_E^2}{r_E^3}$$

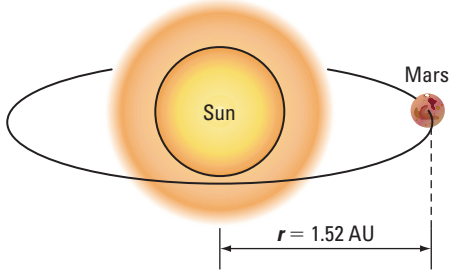
$$= \frac{(1\text{ y})^2}{(1\text{ AU})^3}$$

$$= 1\text{ y}^2/\text{AU}^3$$

The advantage of using the units of an Earth year and astronomical units becomes clear as Kepler's constant works out to 1. Any other planet in our solar system must also have the same constant because it orbits the same focus (the Sun). Be careful not to use this value of Kepler's constant for all systems. For example, if Kepler's third law is used for moons orbiting a planet, or planets orbiting a different sun, the constant will be different.

Example 5.9

Mars has an orbital radius of 1.52 AU (Figure 5.40). What is its orbital period?



▲ **Figure 5.40** Mars has a mean orbital radius of 1.52 AU (not drawn to scale). Kepler's third law can be used to determine its period.

Practice Problems

1. Use Kepler's third law to determine the orbital period of Jupiter. Its orbital radius is 5.203 AU.
2. Pluto takes 90 553 Earth days to orbit the Sun. Use this value to determine its mean orbital radius.
3. A piece of rocky debris in space has a mean orbital distance of 45.0 AU. What is its orbital period?

Analysis and Solution

Start with the equation for determining Kepler's constant:

$$K = \frac{T_{\text{Mars}}^2}{r_{\text{Mars}}^3}$$

where T_{Mars} is the the orbital period of Mars, and r_{Mars} is the mean orbital radius of Mars.

$$T_{\text{Mars}}^2 = Kr_{\text{Mars}}^3$$

$$T_{\text{Mars}} = \sqrt{\left(1 \ \frac{\text{y}^2}{\text{AU}^3}\right)(1.52 \ \text{AU})^3}$$

$$= 1.87 \text{ y}$$

The orbital period of Mars is 1.87 Earth years. In other words, it takes 1.87 Earth years for Mars to go around the Sun once.

Answers

1. 11.87 Earth years
2. 39.465 AU
3. 302 Earth years

A close examination of Table 5.5 shows that as the orbital radius of a planet increases, so does its orbital period. Planets nearest the Sun orbit fastest; that is, they have the shortest years. Planets farthest from the Sun have the longest years.

Kepler's laws don't apply just to planets orbiting the Sun. They apply to all bodies that orbit the same focus in an ellipse. This means that moons orbiting a planet are also subject to Kepler's laws and have their own Kepler's constant. Earth has only one Moon (natural satellite) but Jupiter and Saturn have many, and more are being found all the time. Table 5.6 shows the planets and some of their known moons.

PHYSICS INSIGHT

Kepler's third law can only be applied to bodies orbiting the same object.

▼ Table 5.5 Solar Celestial Bodies

Celestial Body	Mass (kg)	Equatorial Radius (m)	Orbital Period (days)	Mean Orbital Radius (m)	Distance (AU)
Sun	1.99×10^{30}	6.96×10^{8}	—	—	—
Mercury	3.34×10^{23}	2.44×10^{6}	87.97	5.79×10^{10}	0.387
Venus	4.87×10^{24}	6.05×10^{6}	224.7	1.08×10^{11}	0.723
Earth	5.98×10^{24}	6.37×10^{6}	365.24	1.49×10^{11}	1
Mars	6.40×10^{23}	3.40×10^{6}	686.93	2.28×10^{11}	1.524
Ceres	9.5×10^{20}	4.88×10^{5}	1679.8	4.14×10^{11}	2.778
Jupiter	1.90×10^{27}	7.15×10^{7}	4330.6	7.78×10^{11}	5.203
Saturn	5.69×10^{26}	6.00×10^{7}	10 755.7	1.43×10^{12}	9.537
Uranus	8.68×10^{25}	2.59×10^{7}	30 687.2	2.87×10^{12}	19.191
Neptune	1.03×10^{26}	2.48×10^{7}	60 190	4.50×10^{12}	30.069
Pluto	1.20×10^{22}	1.70×10^{6}	90 553	5.91×10^{12}	39.482
Eris	?	$\sim 1.43 \times 10^{6}$	204 540	$\sim 1.01 \times 10^{13}$	67.940
Sedna	4.21×10^{21}	8.5×10^{5}	3 835 020	7.14×10^{13}	479.5

***info* BIT**

As of August 2006, the International Astronomical Union (IAU) finally developed a definition of a planet. To be a planet, a celestial body must orbit a star, be large enough that its own gravity forms it into a spherical shape, and have cleared the neighbourhood around its orbit. Pluto fails to satisfy the last criterion as its orbit is near many other Kuiper belt objects. There are now officially only eight planets in our solar system.

Concept Check

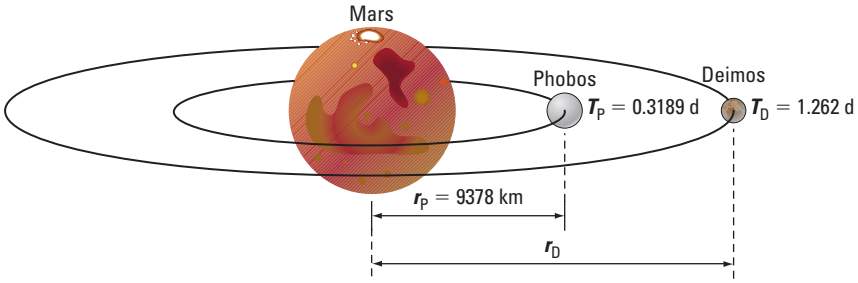
1. What type of orbit would a planet have if the semi-minor axis of its orbit equalled the semi-major axis?
2. Why can't Kepler's third law be applied to Earth's Moon and Jupiter's moon Callisto using the same value for Kepler's constant?
3. Assume astronomers discover a planetary system in a nearby galaxy that has 15 planets orbiting a single star. One of the planets has twice the mass, but the same orbital period and radius as Earth. Could Kepler's constant for our solar system be used in the newly discovered planetary system? Explain.
4. Assume astronomers discover yet another planetary system in a nearby galaxy that has six planets orbiting a single star, but all of the planets' orbits are different from Earth's. Explain how Kepler's third law could apply to this planetary system.
5. Compare planets orbiting a star with points on a rotating solid disc (Figure 5.34).

▼ Table 5.6 The Planets and Their Large Moons

Planet	Moons	Mass (kg)	Equatorial Radius (m)	Orbital Period (Earth days)	Mean Orbital Radius (m)	Eccentricity	Discovered (Year)
Earth	Moon	7.36×10^{22}	1.737×10^{6}	27.322	3.844×10^{8}	0.0549	—
Mars	Phobos	1.063×10^{16}	1.340×10^{4}	0.3189	9.378×10^{6}	0.015	1877
	Deimos	2.38×10^{15}	7.500×10^{3}	1.262	2.346×10^{7}	0.0005	1877
Jupiter (4 most massive)	Io	8.9316×10^{22}	1.830×10^{6}	1.769	4.220×10^{8}	0.004	1610
	Europa	4.79982×10^{22}	1.565×10^{6}	3.551	6.710×10^{8}	0.009	1610
	Ganymede	1.48186×10^{23}	2.634×10^{6}	7.154	1.070×10^{9}	0.002	1610
	Callisto	1.07593×10^{23}	2.403×10^{6}	16.689	1.883×10^{9}	0.007	1610
Saturn (7 most massive)	Mimas	3.75×10^{19}	2.090×10^{5}	0.942	1.855×10^{8}	0.0202	1789
	Enceladus	7×10^{19}	2.560×10^{5}	1.37	2.380×10^{8}	0.00452	1789
	Tethys	6.27×10^{20}	5.356×10^{5}	1.887	2.947×10^{8}	0.00	1684
	Dione	1.10×10^{21}	5.600×10^{5}	2.74	3.774×10^{8}	0.002	1684
	Rhea	2.31×10^{21}	7.640×10^{5}	4.52	5.270×10^{8}	0.001	1672
	Titan	1.3455×10^{23}	2.575×10^{6}	15.945	1.222×10^{9}	0.0292	1655
	Iapetus	1.6×10^{21}	7.180×10^{5}	79.33	3.561×10^{9}	0.0283	1671
Uranus (5 most massive)	Miranda	6.6×10^{19}	2.400×10^{5}	1.41	1.299×10^{8}	0.0027	1948
	Ariel	1.35×10^{21}	5.811×10^{5}	2.52	1.909×10^{8}	0.0034	1851
	Umbriel	1.17×10^{21}	5.847×10^{5}	4.14	2.660×10^{8}	0.005	1851
	Titania	3.53×10^{21}	7.889×10^{5}	8.71	4.363×10^{8}	0.0022	1787
	Oberon	3.01×10^{21}	7.614×10^{5}	13.46	5.835×10^{8}	0.0008	1787
Neptune (3 most massive)	Proteus	5.00×10^{19}	2.080×10^{5}	1.12	1.176×10^{8}	0.0004	1989
	Triton	2.14×10^{22}	1.352×10^{6}	5.8766	3.548×10^{8}	0.000016	1846
	Nereid	2.00×10^{19}	1.700×10^{5}	360.14	5.513×10^{9}	0.7512	1949

Example 5.10

Mars has two moons, Deimos and Phobos (Figure 5.41). Phobos has an orbital radius of 9378 km and an orbital period of 0.3189 Earth days. Deimos has an orbital period of 1.262 Earth days. What is the orbital radius of Deimos?



▲ **Figure 5.41** The orbital period and radius of Phobos can be used with the orbital period of Deimos to determine its orbital radius (not drawn to scale).

Given

$T_P = 0.3189$ d

$r_P = 9378$ km

$T_D = 1.262$ d

Required

orbital radius of Deimos (r_D)

Analysis and Solution

Both Phobos and Deimos orbit the same object, Mars, so Kepler's third law can be used to solve for the orbital radius of Deimos. The units of days and kilometres do not need to be changed to years and astronomical units because Kepler's third law is simply a ratio. It is important to be consistent with the units. If we use the units of kilometres for the measure of orbital radius, then the answer will be in kilometres as well.

$$\frac{T_D^2}{r_D^3} = \frac{T_P^2}{r_P^3}$$

$$r_D^3 = \frac{T_D^2 r_P^3}{T_P^2}$$

$$= \frac{(1.262\ \cancel{\text{d}})^2 (9378\ \text{km})^3}{(0.3189\ \cancel{\text{d}})^2}$$

$$= 1.2916 \times 10^{13}\ \text{km}^3$$

$$r_D = \sqrt[3]{1.2916 \times 10^{13}\ \text{km}^3}$$

$$= 2.346 \times 10^4\ \text{km}$$

Paraphrase

The orbital radius of Deimos is 2.346×10^4 km. This answer is reasonable since a planet with a larger orbital radius than Deimos will also have a larger period.

Practice Problems

1. Titan is one of the largest moons in our solar system, orbiting Saturn at an average distance of 1.22×10^9 m. Using the data for Dione, another moon of Saturn, determine Titan's orbital period.
2. The Cassini-Huygens probe began orbiting Saturn in December 2004. It takes 147 days for the probe to orbit Saturn. Use Tethys, one of Saturn's moons, to determine the average orbital radius of the probe.
3. Astronomers are continually finding new moons in our solar system. Suppose a new moon X is discovered orbiting Jupiter at an orbital distance of 9.38×10^9 m. Use the nearest moon to determine its orbital period.

Answers

1. 15.9 d
2. 5.38×10^6 km
3. 186 d

5-4 Design a Lab

Orbital Period and Radius

Kepler determined that planets have slightly elliptical orbits. He also determined empirically that the period and radius of the planet's orbit were related. The purpose of this lab is to examine the relationship between the period and radius of planets' orbits around the Sun.

The Question

What relationship exists between the orbital period and orbital radius of planets orbiting the Sun?

Design and Conduct Your Investigation

This lab should be designed to investigate the relationship between period and radius. State a hypothesis relating the orbital period to the radius for planets orbiting the Sun. Remember to use an "if/then" statement. The lab should show how a planet's orbital radius affects its orbital period. To do this, use Table 5.5 to select data for at least five planets' periods and orbital radii.

Organize the relevant data from Table 5.5 into a chart in a suitable order that can be used to plot a graph using a graphing calculator or other acceptable means. Choose appropriate units for the manipulated and responding variables.

Look at the type of graph that results to draw a conclusion relating period and radius. Test the relationship you discovered from this lab with the relationship that exists between the orbital radius and period of a planet's moons. Do this by picking a planet from Table 5.6 that has several moons and perform the same procedure and analysis. Compare the relationships of orbital radius and period of the planets with that of the moons. Comment on any similarities or differences.

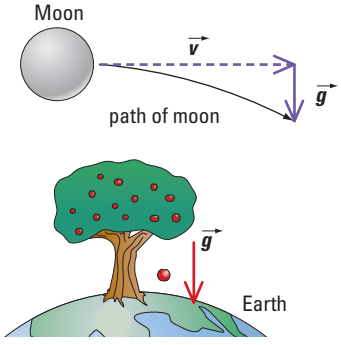
eWEB

In August 2006, the IAU decided to create three classifications of solar system objects: planets, dwarf planets, and small solar system objects. Pluto is now classified as a dwarf planet, along with Eris and Ceres (formerly an asteroid). To be a dwarf planet, the celestial body must orbit the Sun and be large enough that its own gravity forms it into a spherical shape, but not large enough that it has cleared the neighbourhood around its orbit. To learn about dwarf planets, follow the links at: www.pearsoned.ca/school/physicssource.

Newton's Version of Kepler's Third Law

Some stories suggest that Newton was sitting under a tree when an apple fell to the ground and inspired him to discover gravity. This is definitely not what happened, as gravity was already known to exist. But the falling apple did lead him to wonder if the force of gravity that caused the apple to fall could also be acting on the Moon pulling it toward Earth. This revelation might seem obvious but that's only because we have been taught that it's true. In his day, Kepler theorized that magnetism made the Moon orbit Earth, and planets orbit the Sun!

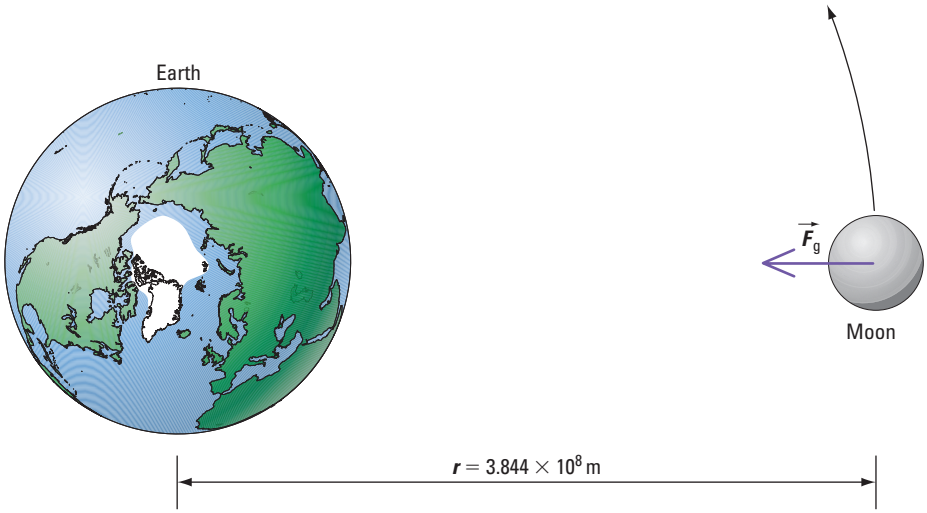
For most of the 1600s, scientists had been trying to predict where planets would be in their orbit at specific times. They failed to grasp the underlying mechanism responsible for the elliptical orbits that Kepler had shown to exist. In 1665, Newton finally recognized what no one else did: the centripetal force acting on the Moon was the force of gravity (Figure 5.42). The Moon was being pulled toward Earth like a falling apple. But the Moon was also moving off tangentially so that the rate at which it was falling matched the rate at which Earth curved away from it. In fact, the same mechanism was responsible for the planets orbiting the Sun.



▲ **Figure 5.42** The Moon is falling to Earth with the acceleration of gravity just like the apple does. But the Moon also has a tangential velocity ($\vec{v}$) keeping it at the same distance from Earth at all times.

By recognizing that the centripetal force and force of gravity were the same, Newton solved the mystery of planetary motion. His derivations mathematically proved Kepler's third law. The implications of Newton's work were huge because scientists now had the mathematical tools necessary to explore the solar system in more depth.

Determining the Speed of a Satellite



▲ **Figure 5.43** The Moon experiences a centripetal force that is the force of gravity of Earth on the Moon.

Two of Newton's derivations deserve close examination. The first derivation uses the orbital radius of a body orbiting a planet or the Sun to determine the body's velocity.

Recall from Chapter 4 that Newton had determined the equation for the force of gravity that one object exerts on another:

$$F_g = \frac{Gm_1m_2}{r^2}$$

He correctly reasoned that the gravitational force exerted by the Moon and Earth must be the centripetal force acting on the Moon (Figure 5.43). So:

$$F_c = F_g$$

If we substitute the equations for centripetal force and gravity into this equation, we obtain:

$$\frac{m_{Moon}v^2}{r} = \frac{Gm_{Moon}m_{Earth}}{r^2}$$

where G is the universal gravitational constant, m_{Moon} is the mass of the Moon in kilograms, m_{Earth} is the mass of Earth in kilograms, v is the speed of the Moon in metres per second, and r is the orbital radius of the Moon in metres.

info BIT

Pluto and its moon Charon are close enough in mass that they have a common centre of gravity between them in space (see Extrasolar Planets on page 283). Charon does not orbit Pluto — both bodies orbit their common centre of gravity. Since their centre of gravity is in space and not below the surface of Pluto, they form a binary system. Astronomers have been aware of binary stars in our universe for many years (i.e., two stars that have a common centre of gravity in space around which they revolve). Pluto and Charon are unique because they are the only planet-moon system like this in our solar system.

PHYSICS INSIGHT

Equation 13 uses the mass of Earth, but is not restricted to it. In a more general sense, the mass refers to the object being orbited.

The mass of the Moon cancels, leaving:

$$\frac{v^2}{r} = \frac{Gm_{\text{Earth}}}{r^2}$$

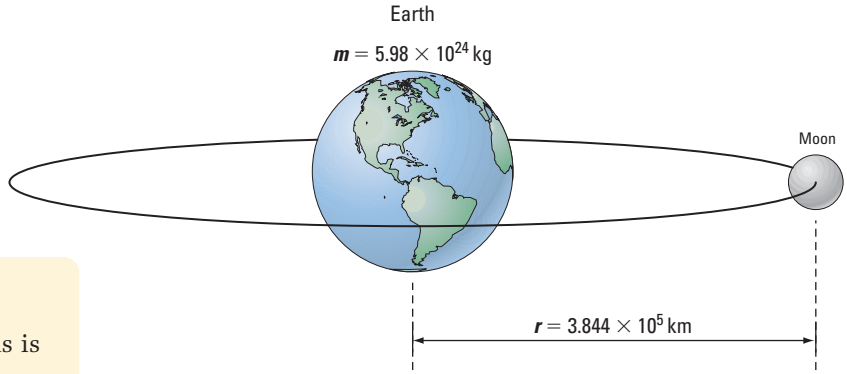
Solving for v gives:

$$v = \sqrt{\frac{Gm_{\text{Earth}}}{r}} \tag{13}$$

In its current form, this equation determines the speed of any object orbiting Earth. The speed of an object orbiting any planet, the Sun, or another star for that matter, can be determined by using the mass of the object being orbited in place of the mass of Earth.

Example 5.11

Earth's Moon is 3.844×10^5 km from Earth (Figure 5.44). Determine the orbital speed of the Moon.



▲ **Figure 5.44** The orbital speed of the Moon can be determined from its orbital radius and the mass of Earth.

Analysis and Solution

Convert the radius of the Moon's orbit to SI units. Then use the mass of Earth in equation 13.

$$r = (3.844 \times 10^5 \text{ km}) \times \frac{1000 \text{ m}}{\text{km}}$$

$$= 3.844 \times 10^8 \text{ m}$$

$$v = \sqrt{\frac{Gm_{\text{Earth}}}{r}}$$

$$= \sqrt{\frac{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24} \text{ kg})}{3.844 \times 10^8 \text{ m}}}$$

$$= 1.02 \times 10^3 \text{ m/s}$$

The orbital speed of the Moon is 1.02×10^3 m/s or 3.67×10^3 km/h.

Practice Problems

1. Neptune's average orbital radius is 4.50×10^{12} m from the Sun. The mass of the Sun is 1.99×10^{30} kg. What is Neptune's orbital speed?
2. The moon Miranda orbits Uranus at a speed of 6.68×10^3 m/s. Use this speed and the mass of Uranus to determine the radius of Miranda's orbit. The mass of Uranus is 8.68×10^{25} kg.

Answers

1. 5.43×10^3 m/s
2. 1.30×10^8 m

Measuring the Orbital Height of a Satellite

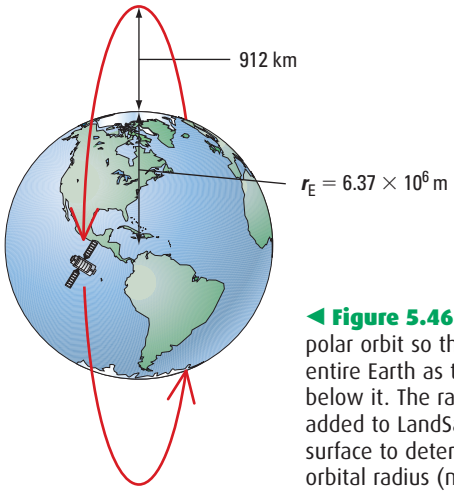
Recall from Chapter 4 that the radius is always measured from the centre of one object to the centre of the other. This means that the orbital radius refers to the distance from centre to centre when measuring the distance from Earth to the Moon or from the Sun to the planets. Figure 5.45 shows the Earth-Moon system drawn to scale.

For an artificial (human-made) satellite, the height of its orbit is usually measured from Earth's surface. To determine the velocity of an artificial satellite, you must first find its proper orbital height (from the centre of Earth). This is done by adding Earth's radius to the height of the satellite above Earth's surface.

▲ **Figure 5.45** The Earth-Moon system drawn to scale. The radius of Earth doesn't need to be considered when comparing the distance between Earth and the Moon because it is insignificant compared to the great distance separating the two bodies. The distance between Earth and a communication satellite (35 880 km away) is small, so Earth's radius must be included in calculations involving orbital radius and period.

Example 5.12

LandSat is an Earth-imaging satellite that takes pictures of Earth's ozone layer and geological features. It orbits Earth at the height of 912 km (Figure 5.46). What are its orbital speed and its period?



◀ **Figure 5.46** LandSat follows a polar orbit so that it can examine the entire Earth as the planet rotates below it. The radius of Earth must be added to LandSat's height above the surface to determine the satellite's orbital radius (not drawn to scale).

Given

height of the satellite above Earth's surface = 912 km

Required

LandSat's orbital speed (v) and period (T)

Practice Problems

1. The International Space Station orbits Earth at a height of 359.2 km. What is its orbital speed?
2. The Chandra X-ray satellite takes X-ray pictures of high-energy objects in the universe. It is orbiting Earth at an altitude of 114 593 km. What is its orbital period?

Answers

1. 7.70×10^3 m/s
2. 4.19×10^5 s

Analysis and Solution

The radius of LandSat's orbit must be measured from the centre of Earth. To do this, add Earth's radius to the satellite's height above the planet's surface. Then determine the speed and period of the satellite.

$$\begin{aligned} r &= r_{\text{Earth}} + 912\,000 \text{ m} \\ &= (6.37 \times 10^6 \text{ m}) + (9.12 \times 10^5 \text{ m}) \\ &= 7.28 \times 10^6 \text{ m} \end{aligned}$$

$$F_c = F_g$$

$$\frac{m_{\text{Moon}} v^2}{r} = \frac{G m_{\text{Moon}} m_{\text{Earth}}}{r^2}$$

$$v = \sqrt{\frac{G m_{\text{Earth}}}{r}}$$

$$= \sqrt{\frac{\left(6.67 \times 10^{-11} \dfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24} \text{ kg})}{7.28 \times 10^6 \text{ m}}}$$

$$= 7.40 \times 10^3 \text{ m/s}$$

$$T = \frac{2\pi r}{v}$$

$$= \frac{2\pi(7.28 \times 10^6 \text{ m})}{7.40 \times 10^3 \dfrac{\text{m}}{\text{s}}}$$

$$= 6.18 \times 10^3 \text{ s}$$

Paraphrase

The speed of the satellite is 7.40×10^3 m/s (26 640 km/h). It orbits Earth once every 6.18×10^3 s (103 minutes).

Determining the Mass of a Celestial Body

A second derivation from Newton's version of Kepler's third law has to do with determining the mass of a planet or the Sun from the period and radius of a satellite orbiting it.

For any satellite in orbit around a planet, you can determine its speed if you know the mass of the planet. But just how do you determine the mass of the planet? For example, how can you "weigh" Earth? Newton realized that this was possible. Let's look at the equality $F_c = F_g$ again, but this time we will use equation 8 for centripetal force. Recall that:

$$F_c = \frac{4\pi^2 m_{\text{Moon}} r}{T^2_{\text{Moon}}}$$

where m_{Moon} is the mass of the Moon in kilograms, and T_{Moon} is the orbital period of the Moon in seconds.

eTECH

For an interactive simulation of the effect of a star's mass on the planets orbiting it, follow the links at www.pearsoned.ca/physicssource.

Since $F_c = F_g$, then

$$\frac{4\pi^2 m_{Moon} r}{T^2_{Moon}} = \frac{G m_{Moon} m_{Earth}}{r^2}$$

Solve for m_{Earth}:

$$m_{Earth} = \frac{4\pi^2 r^3}{T^2_{Moon} G} \quad (14)$$

You can determine Earth's mass by using the orbital radius and period of its satellite, the Moon. From Table 5.6 on page 274, the Moon's period and radius are:

- period of the Moon (T_{Moon}) = 27.3 days or 2.36×10^6 s
- radius of the Moon's orbit (r_{Moon}) = 384 400 km or 3.844×10^8 m

$$m_{Earth} = \frac{4\pi^2 (3.844 \times 10^8 \text{ m})^3}{(2.36 \times 10^6 \text{ s})^2 \left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)}$$

$$= 6.04 \times 10^{24} \text{ kg}$$

The mass of Earth is 6.04×10^{24} kg, which is close to the accepted value of 5.98×10^{24} kg.

Of course, this equation is not restricted to the Earth-Moon system. It applies to any celestial body that has satellites. For example, the Sun has eight planets that are natural satellites. Any one of them can be used to determine the mass of the Sun.

PHYSICS INSIGHT

Equation 14 uses the mass of Earth and the period of its Moon. This equation can be used for any celestial body with a satellite orbiting it. In a more general sense, the mass refers to the object being orbited, and the period is the period of the orbiting object.

Concept Check

1. What insight did Newton have that helped him explain the motion of the planets?
2. If Newton were told that our solar system is orbiting the centre of our galaxy, how would he explain this?
3. What previously immeasurable quantity was Newton able to determine once he developed his version of Kepler's third law?

Orbital Perturbations

At about the same time as Kepler was figuring out the mechanism of the solar system, Galileo Galilei (1564–1642) pointed a relatively new invention at the sky. He began using a telescope to closely examine Jupiter. Only a few planets are visible to the naked eye: Mercury, Venus, Mars, Saturn, and Jupiter. Until the early 1600s, any observations of these planets were done without the aid of a telescope. Within a few months of using only an 8-power telescope, Galileo had discovered four moons of Jupiter. It quickly became apparent how useful a telescope would be in the field of astronomy.

e WEB

To learn more about Galileo's discoveries with the telescope in 1609 and 1610, follow the links at www.pearsoned.ca/school/physicssource.

Within the space of 100 years, telescope technology improved dramatically, and the field of astronomy began its golden age. Astronomers plotted the positions of the planets more accurately than ever before and could peer deeper into the solar system. William Herschel (1738–1822) discovered the new planet Uranus in 1781, which created enormous interest. However, it wasn't long before astronomers noticed something strange about the orbit of Uranus. The orbital path of Uranus deviated from its predicted path slightly, just enough to draw attention. Astronomers called this deviation, or disturbance, an **orbital perturbation**.

orbital perturbation: irregularity or disturbance in the predicted orbit of a planet

The Discovery of Neptune

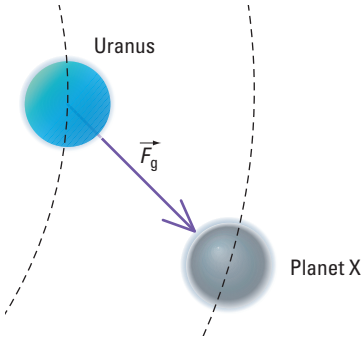
It had been over 120 years since Kepler and Newton had developed the mathematical tools necessary to understand and predict the position of the planets and their moons. Confident in the reliability of these laws, astronomers looked for a reason for the perturbation in the orbit of Uranus. According to mathematical predictions, Uranus should have been farther along in its orbit and closer to the Sun than it actually was. Somehow its progress was being slowed, and it was being pulled away from the Sun.

Recall that anything with mass creates a gravitational field. The strength of this field depends on the mass of the object and the separation distance from it. The orbit of Uranus was minutely perturbed. Could another as-yet-undiscovered planet be exerting a gravitational pull or tug on Uranus whenever the orbital path took these two planets close together? If there was a planet X farther out and behind Uranus, it would exert a gravitational pull that would slow Uranus down and pull its orbit outward (Figure 5.47). This could explain the perturbation in the orbit of Uranus and was precisely the assumption that two astronomers, Urbain Le Verrier (1811–1877) of France and John Adams (1819–1892) of Britain, made in 1845.

By examining exactly how much Uranus was pulled from its predicted position, Le Verrier and Adams could use Newton's law of gravitation to mathematically predict the size and position of this mysterious planet — if it existed. Working independently, both scientists gave very similar predictions of where to look for the planet. In September 1846, at the request of Le Verrier, German astronomer Johann Gottfried Galle (1812–1910) looked for the planet where Le Verrier predicted it would be and he found it! Le Verrier called the planet Neptune. It remained the solar system's outermost planet for the next 84 years until astronomers discovered Pluto in 1930 by analyzing the orbital perturbations of Neptune. (In 2006, Pluto was reclassified as a dwarf planet.)

eWEB

Telescopes and space technologies are continually improving, and new planetary objects are being found all the time. For the most up-to-date list of planets and celestial bodies in our solar system, follow the links at www.pearsoned.ca/school/physicssource.



▲ **Figure 5.47** If a planet X existed and was behind Uranus in its orbit, it would pull Uranus back and outward.

The Search for Other Planets

The search for more planets in our solar system continues. A large band of rocky debris called the Kuiper belt lies 30 to 50 AU from the Sun. Pluto is a resident in this belt. Many scientists believed that Pluto was unlikely to be the only large resident of the belt and that the belt very likely contained more planet-sized objects.

In July 2005, a Kuiper belt object about 1.5 times bigger than Pluto was found. The scientific community has now given it the status of a dwarf planet, but at the time of publication of this book, it did not yet have an official name. Do other larger-than-Pluto bodies exist in the Kuiper belt? Probably, but their low reflectivity and extreme distance make them difficult to detect.

THEN, NOW, AND FUTURE — Extrasolar Planets

For many centuries, humans have wondered if they were the only intelligent life in the universe. It seems unlikely, given the multitude of stars in our galaxy alone. However, for any life to exist, it's pretty safe to assume that it must have a planet to inhabit. Until 1995, there was no conclusive proof that any other star besides our Sun had planets.

In October 1995, the first extrasolar planet was found orbiting a star similar in size to our Sun. It is called "extrasolar" because it is outside ("extra") our solar system. This planet was named 51 Pegasi b after the star around which it revolves (Figure 5.48).

It is huge but its orbital radius is extremely small. It is unlikely to support life as its gravitational field strength and temperature are extreme. Yet the discovery was a milestone since finding a planet orbiting a bright star 48 light years away is a very difficult task. A planet doesn't produce any light of its own, and it would be relatively dark compared to the very bright star beside it.

The light coming from a star is so bright it makes direct observation of a nearby planet difficult. Imagine staring into a car's headlight on a dark night and trying to see an ant crawling on the bumper.

Now imagine the car is 23 000 km away, and you get an idea of the magnitude of the problem. However, the increasing power of telescopes and innovative detection techniques are yielding new planet findings all the time.

How is this accomplished? There are several different ways, but the most common and productive way is not to look for the planet directly but to look at the star that it is orbiting and watch for perturbations (also called wobble) in the star's movement.

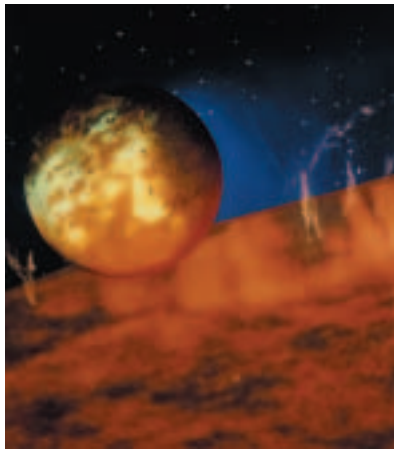
That's right, stars move. If they don't have a planet or planets orbiting them, then they simply move in a linear fashion through space. If they have a planet or planets in orbit around them, not only do they move linearly, but they also wobble in a circular path. This is because a planet exerts a gravitational pull on its star just as the star exerts the same pull on the planet.

They both revolve around each other, just like you and a very heavy object would if you spun the object around in a circle at the end of a rope. Since the star is much more massive than the planet, its orbital radius is very small. This perturbation of the star is detectable and is indirect evidence that a planet must be orbiting it.

As you can imagine, for a star to have a noticeable wobble, the planet that orbits it must be relatively large and fairly close to the star. This would make conditions on the planet inhospitable for life as we know it. But the search goes on and as our technology improves, who knows what we may find?

Questions

1. Why is it so hard to detect an extrasolar planet?
2. What new technologies and techniques are making it possible to detect extrasolar planets?
3. Why is it unlikely that any life exists on the extrasolar planets discovered so far?



▲ **Figure 5.48** An artist's conception of 51 Pegasi b — the first extrasolar planet found orbiting a star similar to our own.

▲ **Figure 5.49** A satellite in low Earth orbit

Artificial Satellites

At present, there are well over 600 working artificial satellites orbiting Earth. About half of them are in a low or medium Earth orbit, ranging from 100 to 20 000 km above Earth's surface (Figure 5.49). The other half are geostationary satellites that orbit Earth at a distance of precisely 35 880 km from Earth's surface, directly above the equator. Depending on their design and orbit, satellites perform a variety of tasks. Weather, communication, observation, science, broadcast, navigation, and military satellites orbit Earth at this moment. These include a ring of GPS (global positioning system) satellites that are used to triangulate the position of a receiver wherever it may be. With the help of a GPS receiver, you could find your position on the planet to within a few metres.

All satellites are designed to receive information from and transmit information to Earth. Each satellite has an antenna that is used to receive radio signals from satellite stations on the ground. At the same time, satellites send information back down to Earth for people to use.

5-5 Decision-Making Analysis

Required Skills
- Initiating and Planning
- Performing and Recording
- Analyzing and Interpreting
- Communication and Teamwork

The Costs and Benefits of Putting a Satellite into Orbit

The Issue

Satellites perform a variety of tasks that make them almost indispensable. They map Earth, find geological formations and minerals, help us communicate over great distances and in remote areas, and help predict the movement of weather systems such as hurricanes. Opponents of the continued unregulated use of satellites argue that the cost of deploying satellites is enormous; they don't have a long lifespan; and their failure rate is high. Furthermore, once a satellite is in a medium or geostationary orbit, it will stay there even after it becomes inoperative. It turns into nothing more than very expensive and dangerous space junk.

Background Information

The first satellite in orbit was Sputnik in 1957. Since that time there have been over 4000 launches, and space has become progressively more crowded. The best estimates suggest that there are about 600 active satellites in orbit, and about 6000 pieces of space debris that are being tracked. The space debris can be very hazardous for missions carrying astronauts to low Earth orbit. If hit by even a small piece of orbiting debris, a spacecraft could be destroyed.

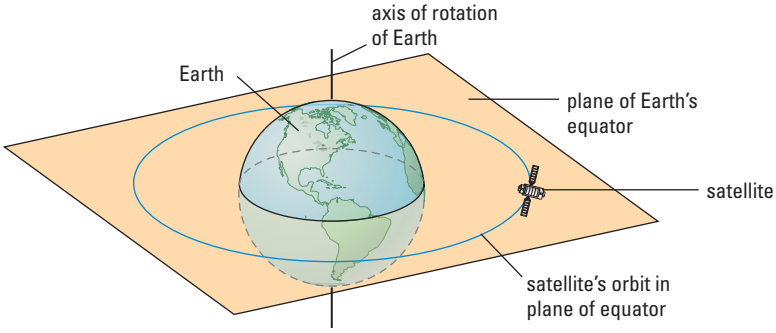
To limit the overcrowding of space, satellite manufacturers are designing more sophisticated satellites that can handle a higher flow of information so that fewer satellites have to be deployed. This drives up the cost of manufacturing satellites because they are more technologically advanced than their predecessors. Unfortunately, as well as being more sophisticated, they are more prone to failure.

Analyze and Evaluate

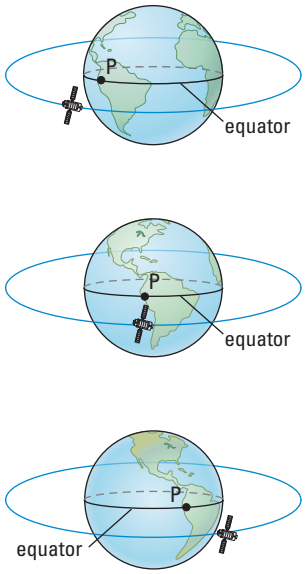
1. Identify two different types of satellites based on the type of job they perform.
2. For each type of satellite from question 1:
   (a) Identify the type of orbit that it occupies and explain the job that it performs.
   (b) Determine the approximate cost of deployment (getting it into space).
   (c) Determine its expected lifespan.
3. Suggest an alternative technology that could be used in place of each of these satellites. Analyze the effectiveness and cost of this technology compared with the satellite.
4. Propose possible changes that could be made to the way satellites are built and deployed that could lessen the overcrowding of space.

Geostationary Satellites

Geostationary satellites may be the most interesting of all satellites because they appear to be stationary to an observer on Earth's surface, even though they travel around Earth at high velocity. They are placed at a specific altitude so that they make one complete orbit in exactly 24 h, which is the same as Earth's rotational period (Figure 5.50). These satellites are placed in the plane of the equator, so they will have exactly the same axis of rotation as Earth, and will stay fixed over the same spot on the planet (Figure 5.51). To an observer on the ground, geostationary satellites appear motionless.



▲ **Figure 5.50** Geostationary satellites orbit in the plane of the equator with the same axis of rotation as Earth.



▲ **Figure 5.51** A geostationary satellite moves with a fixed point (P) on the Earth since both revolve around the same axis with the same period. To an observer on the ground, the satellite does not appear to be moving.

Communication satellites are geostationary. A communication signal, such as a telephone or TV signal, can be sent from the ground at any time of the day to the nearest geostationary satellite located over the equator. That satellite can then relay the signal to other geostationary satellites located over different spots on Earth, where the signal is then transmitted back down to the nearest receiving station.

Weather satellites also make use of this orbit. They may be "parked" near a landmass such as North America, using cameras to map the weather. Weather forecasters receive a continuous stream of information from the satellites that allows them to predict the weather in their area.

This type of orbit is in high demand and is filled with satellites from many different countries. Unfortunately, the orbit must be fixed at 35 880 km from Earth's surface and be directly over the equator. This orbit risks being overcrowded. If satellites are placed too close together, their signals can interfere with each other. Satellites also tend to drift slightly in their orbit and therefore cannot be placed close to each other. Many derelict satellites that were placed in geostationary orbits are still there taking up room, since they continue to orbit even after they no longer function. The limited room available is filling up fast. At present, about half (at least 300) of the active satellites in orbit are geostationary.

An artificial satellite obeys the same laws of physics as a natural satellite does. The orbital radius determines its orbital period and speed, and it is governed by the same laws that describe the motion of moons around a planet or planets around the Sun. That means low Earth orbit satellites make one complete orbit in about 90 min, while geostationary satellites take exactly one day.

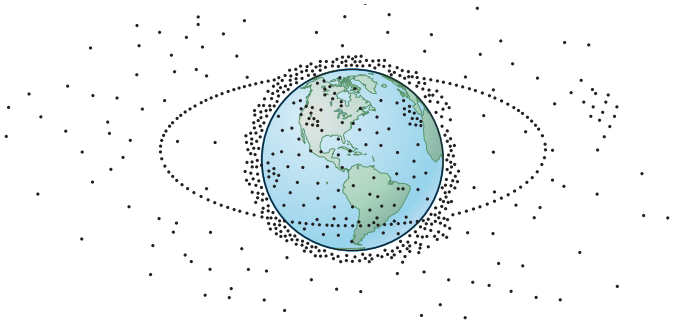
info BIT

Geostationary satellites are also referred to as geosynchronous satellites.

e WEB

To learn more about real-time tracking of satellites in orbit around Earth, follow the links at www.pearsoned.ca/school/physicssource.

In the short space of time since the first satellite, Sputnik, was launched in October 1957, satellites have become indispensable (Figure 5.52). They have enabled us to see areas of our planet and atmosphere never seen before. They have enabled us to know our location anywhere on Earth within a few metres, and have allowed us to communicate with the remotest places on Earth. The future of satellites seems assured.



▶ **Figure 5.52** Around Earth, space is crowded with the multitude of satellites currently in orbit. The large ring farthest from Earth is made up of geostationary satellites.

5.3 Check and Reflect

Knowledge

1. What is an astronomical unit?
2. Why is the orbital radius of a planet not constant?
3. An eccentricity of 0.9 indicates what kind of shape?
4. Where in a planet's orbit is its velocity the greatest? The smallest? Why?
5. State the condition necessary for Kepler's third law to be valid.
6. Is Kepler's constant the same for moons orbiting a planet as it is for planets orbiting the Sun? Why?
7. Does our Sun experience orbital perturbation? Explain.

Applications

8. Sketch a graph that shows the trend between the planets' orbital radius and their period.
9. Venus has an average orbital period of 0.615 years. What is its orbital radius?
10. Another possible planet has been discovered, called Sedna. It has an average orbital radius of 479.5 AU. What is its average orbital velocity?
11. Jupiter's moon Io has an orbital period of 1.769 d and an average orbital radius of 422 000 km. Another moon of Jupiter, Europa, has an average orbital radius of 671 000 km. What is Europa's orbital period?
12. Determine the average orbital speed of Mimas using orbital data from Table 5.6 on page 274.
13. Using the orbital period and radius of Venus from Table 5.5 on page 273, determine the mass of the Sun.

Extensions

14. As more satellites are put into space, the amount of orbital debris increases. What solutions can you suggest to decrease the amount of space junk?
15. Using a graphing calculator or other suitable software, plot a graph of velocity versus radius for an artificial satellite orbiting Earth.

eTEST

To check your understanding of satellites and celestial bodies in circular motion, follow the eTest links at www.pearsoned.ca/school/physicssource.

Key Terms and Concepts

axle
axis of rotation
uniform circular motion
centripetal acceleration
centripetal force
cycle
revolution
period
frequency
rpm
Kepler's laws
ellipse
eccentricity
orbital period
orbital radius
satellite
artificial satellite
orbital perturbation
extrasolar planet

Key Equations

$$v = \frac{2\pi r}{T}$$

$$a_c = \frac{4\pi^2 r}{T^2}$$

$$\frac{T_a^2}{r_a^3} = \frac{T_b^2}{r_b^3}$$

$$a_c = \frac{v^2}{r}$$

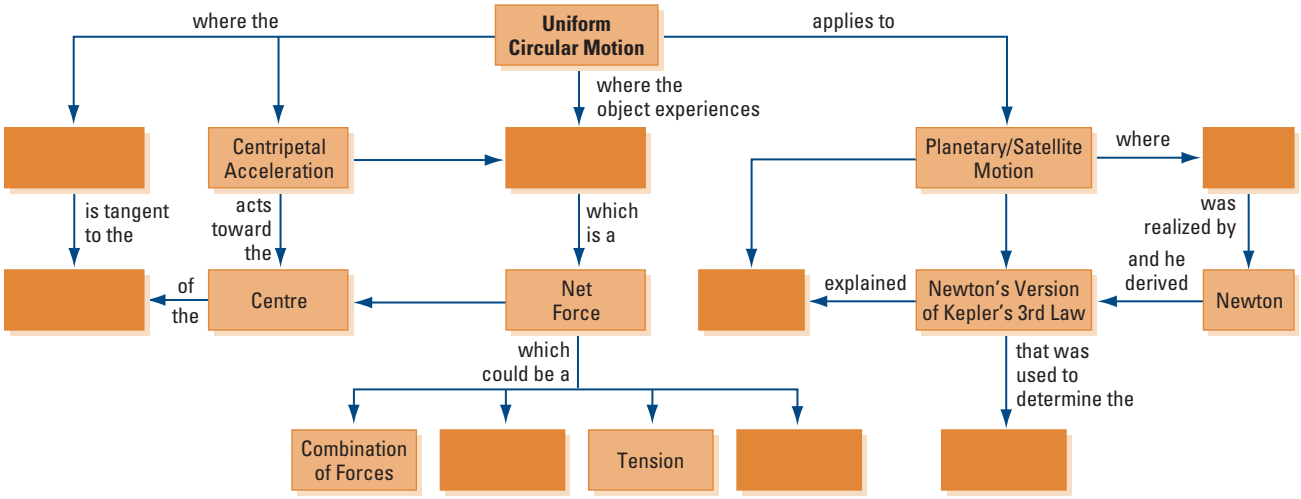
$$F_c = \frac{4\pi^2 mr}{T^2}$$

$$F_c = \frac{mv^2}{r}$$

$$\frac{T_a^2}{r_a^3} = K$$

Conceptual Overview

The concept map below summarizes many of the concepts and equations in this chapter. Copy and complete the map to have a full summary of the chapter.



▲ **Figure 5.53**

Knowledge

1. (5.1) What is the direction of centripetal acceleration?

2. (5.1) Describe what produces the centripetal force in the following cases:

(a) A boat makes a turn.

(b) A plane makes a horizontal turn.

(c) A satellite orbits Earth.

3. (5.1) What force is responsible for the tug your hand feels as you spin an object around at the end of a rope?

4. (5.1) Sketch a diagram of a mass moving in a vertical circle. Draw the velocity, centripetal acceleration, and centripetal force vectors at a point on the circle's circumference.

5. (5.2) Why do spinning tires tend to stretch?

6. (5.2) A heavy mass attached to the end of a cable is spinning in a vertical circle. In what position is the cable likely to break?

7. (5.2) What force acts as the centripetal force for a motorcycle making a turn?

8. (5.2) Explain why a truck needs a larger turning radius than a car when they move at the same speed.

9. (5.3) In what position of its orbit does a planet move the fastest?

10. (5.3) In the orbit of a planet, what does the semi-major axis of an ellipse represent?

11. (5.3) Equation 14 can be confusing because it uses the mass of one object and the period of another.

(a) Explain what m_{Earth} and T_{Moon} represent.

(b) Explain how equation 14 is used in the most general case.

12. (5.3) Use Kepler's second law to explain why planets move more quickly when they are nearer to the Sun than when they are farther away in their orbital path.

13. (5.3) Why can't Kepler's constant of 1 y^2/AU^3 be used for moons orbiting planets or planets in other solar systems?

Applications

14. If the frequency of a spinning object is doubled, what effect does this have on the centripetal acceleration?

15. A slingshot containing rocks is spun in a vertical circle. In what position must it be released so that the rocks fly vertically upward?

16. People of different masses are able to ride a roller coaster and go through a vertical loop without falling out. Show the mathematical proof of this.

17. An eagle circles above the ground looking for prey. If it makes one complete circle with the radius of 25.0 m in 8.0 s, what is its speed?

18. A ride at the fair spins passengers around in a horizontal circle inside a cage at the end of a 5.0-m arm. If the cage and passengers have a speed of 7.0 m/s, what is the centripetal force the cage exerts on an 80.0-kg passenger?

19. What is the minimum speed that a glider must fly to make a perfect vertical circle in the air if the circle has a radius of 200.0 m?

20. A child spins an 800.0-g pail of water in a vertical circle at the end of a 60.0-cm rope. What is the tension of the rope at the top of the swing if it is spinning with the frequency of 2.0 Hz?

21. A driver is negotiating a turn on a mountain road that has a radius of 40.0 m when the 1600.0-kg car hits a patch of ice. The coefficient of friction between the ice and the wheels is 0.500. If the car is moving at 30.0 km/h, determine if it skids off the road.

22. The blade of a table saw has a diameter of 25.4 cm and rotates with a frequency of 750 rpm. What is the centripetal acceleration at the edge of the blade?

23. The tip of a propeller on an airplane has a radius of 0.90 m and experiences a centripetal acceleration of 8.88×10^4 m/s^2. What is the frequency of rotation of the propeller in rpm?

24. Using information and equations from this chapter, verify that the speed of Earth in orbit around the Sun is approximately 107 000 km/h.

25. A car rounds a corner of radius 25.0 m with the centripetal acceleration of 6.87 m/s^2 and a hubcap flies off. What is the speed of the hubcap?

26. An electron ($m = 9.11 \times 10^{-31}$ kg) caught in a magnetic field travels in a circular path of radius 30.0 cm with a period of 3.14×10^{-8} s.

(a) What is the electron's speed?

(b) What is the electron's centripetal acceleration?

27. Halley's comet has an orbital period of 76.5 y. What is its mean orbital radius?

28. An extrasolar planet is found orbiting a star in the Orion nebula. Determine the mass of the star if the planet has an orbital period of 400.0 Earth days and an orbital radius of 1.30×10^{11} m.

29. The Milky Way is a spiral galaxy with all the stars revolving around the centre where there is believed to be a super-massive black hole. The black hole is 2.27×10^{20} m from our Sun, which revolves around it at a speed of 1234 m/s.

(a) What is the period of our solar system's orbit around the black hole?

(b) How massive is the black hole?

(c) What centripetal acceleration does our solar system experience as a result of the black hole's gravity?

30. Newton hypothesized that a cannonball fired parallel to the ground from a cannon on the surface of Earth would orbit Earth if it had sufficient speed and if air friction were ignored.

(a) What speed would the cannonball have to have to do this?

(b) From the speed determined in part (a), determine the mass of Earth.

(c) If the mass of the cannonball were doubled, what time would it take to orbit Earth once?

31. Use the Moon's period (27.3 d), its mass, and its distance from Earth to determine its centripetal acceleration and force.

32. Neptune has a moon Galatea that orbits at an orbital radius of 6.20×10^7 m. Use data from Table 5.6 on page 274 to determine Galatea's orbital period.

33. Determine the orbital speed of Ariel, a moon of Uranus, using Table 5.5 (page 273) and Table 5.6 (page 274).

34. A star in the Andromeda galaxy is found to have a planet orbiting it at an average radius of 2.38×10^{10} m and an orbital period of 4.46×10^4 s.

(a) What is the star's mass?

(b) A second planet is found orbiting the same star with an orbital period of 6.19×10^6 s. What is its orbital radius?

Extensions

35. Paraphrase two misconceptions about centripetal force mentioned in this book. How do these misconceptions compare with your preconceptions of centripetal force?

36. Explain why a person standing at the equator weighs less than the same person standing at the North or South Pole.

Consolidate Your Understanding

Create your own summary of uniform circular motion, Kepler's laws, and planetary and satellite motion by answering the questions below. If you want to use a graphic organizer, refer to Student References 4: Using Graphic Organizers pp. 869–871. Use the Key Terms and Concepts listed above and the Learning Outcomes on page 240.

1. In a few sentences, summarize how frequency, period, and velocity affect centripetal acceleration.

2. Explain to a classmate in writing why the velocity at different radii on a spinning disc will vary while the rotational frequency remains constant.

Think About It

Review your answers to the Think About It questions on page 241. How would you answer each question now?

eTEST

To check your understanding of circular motion, follow the eTest links at www.pearsoned.ca/school/physicssource.